Multiscale Modeling in Granular Flow

by

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Abstract

Granular materials are common in everyday experience, but have long-resisted a complete theoretical description. Here, we consider the regime of slow, dense granular flow, for which there is no general model, representing a considerable hurdle to industry, where grains and powders must frequently be manipulated.

Much of the complexity of modeling granular materials stems from the discreteness of the constituent particles, and a key theme of this work has been the connection of the microscopic particle motion to a bulk continuum description. This led to development of the "spot model", which provides a microscopic mechanism for particle rearrangement in dense granular flow, by breaking down the motion into correlated group displacements on a mesoscopic length scale. The spot model can be used as the basis of a multiscale simulation technique which can accurately reproduce the flow in a large-scale discrete element simulation of granular drainage, at a fraction of the computational cost. In addition, the simulation can also successfully track microscopic packing signatures, making it one of the first models of a flowing random packing.

To extend to situations other than drainage ultimately requires a treatment of material properties, such as stress and strain-rate, but these quantities are difficult to define in a granular packing, due to strong heterogeneities at the level of a single particle. However, they can be successfully interpreted at the mesoscopic spot scale, and this information can be used to directly test some commonly-used hypotheses in modeling granular materials, providing insight into formulating a general theory.

Thesis Supervisor: Martin Z. Bazant
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Throughout my time at MIT, **Arshad Kudrolli** from Clark University provided a brilliant experimental perspective, that gave an oft-needed counter-balance to the atmosphere in the applied math department. I would also like to thank my other qualifiying and thesis committee members **Ruben Rosales**, **Dionisios Margetis**, and **Jean-Christophe Nave** for their numerous helpful suggestions. **Daniel Freedman** provided a lot of helpful advice as my first-year academic mentor, and was largely responsible for my original decision to come to MIT. The applied math administrative assistant, **Shirley Entzminger**, frequently went out of her way to help me out with organizational difficulties. Finally, I would like to thank **Jon Wilkening** at UC Berkeley and **Jim Langer** at UC Santa Barbara who provided much support and encouragement in my final months at MIT, as I brought this thesis to a close, and considered future directions.

I arrived in August 2002 with some trepidation, since it was my first time living in the US, and I knew hardly anyone. However, within several weeks I had made many excellent friends through social events at Tang Hall, including **Vikas Anant**, **Julia Cline**, **Natalija Jovanovic**, **Alexandra Kern**, **Karen Lee**, **Vivian Lei**, **Song-**

Hee Paik, **Mitch Peabody**, **Johnna Powell**, **Sarah Rodriguez**, **Kaity Ryan**, **Ming Tang**, **Katy Thorn**, and **Bruce Wu**. Although many of these people moved away over the following years, they formed the bedrock of my social experiences at MIT, and I could write pages about the many great times that we shared. Also members of this group were **Tyrone Hill**, **Stephen Kohen**, and **Shawn Kuo** who later became my roommates for four years at 374 Washington Street, and helped make it one of the most friendly and relaxed environments that any MIT graduate student could hope for.

As time went by in the first year, I also formed numerous other friendships. While still living in Tang Hall, I became close friends with **Miranda Newbery** and **Josep Dorca Luque** with whom I shared a British connection, and with whom I could share a laugh with about the American pronunciation of "tomato". Around that time, I also met **Tony Lau**, who makes truly awesome smoothies, and who convinced me that California really is the place to be. In the winter of 2003, I went sledding with **Hong Ma**, and subsequently enjoyed many crazy adventures with him, whether it was swimming down New Hampshire rivers, running around Manhattan solving puzzles in the middle of the night, or rowing home-made rafts along the Charles River.

At the start of my second year, I met the incredible **Emma Brunskill**, who became my running partner for four years; she was always overflowing with ideas on every subject, and she was the first to cultivate in me a deep appreciation for the many excellent ice cream stores in Cambridge. Several months later, I befriended **Maggie Lee** and **Rachel Sugal** from UMass Amherst, who frequently provided me with an interesting perspective outside the Boston and Cambridge bubble. In the summer, while hiking in Acadia, I met **Valentina Urbanek** who has been a great friend during my time at MIT. Around that time, I also met **Olivia Cheo**, who makes some of the best chocolate cookies that I have ever tasted, and **Caterina Schweidenback**, who is always overflowing with energy and enthusiasm. I went on many excellent hikes and attended a lot of dinner parties with my friends **Marcus Roper** and **Silas Alben** from Harvard University.

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