

Multiscale Modeling in Granular Flow

by

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Abstract

Granular materials are common in everyday experience, but have long-resisted a complete theoretical description. Here, we consider the regime of slow, dense granular flow, for which there is no general model, representing a considerable hurdle to industry, where grains and powders must frequently be manipulated.

Much of the complexity of modeling granular materials stems from the discreteness of the constituent particles, and a key theme of this work has been the connection of the microscopic particle motion to a bulk continuum description. This led to development of the “spot model”, which provides a microscopic mechanism for particle rearrangement in dense granular flow, by breaking down the motion into correlated group displacements on a mesoscopic length scale. The spot model can be used as the basis of a multiscale simulation technique which can accurately reproduce the flow in a large-scale discrete element simulation of granular drainage, at a fraction of the computational cost. In addition, the simulation can also successfully track microscopic packing signatures, making it one of the first models of a flowing random packing.

To extend to situations other than drainage ultimately requires a treatment of material properties, such as stress and strain-rate, but these quantities are difficult to define in a granular packing, due to strong heterogeneities at the level of a single particle. However, they can be successfully interpreted at the mesoscopic spot scale, and this information can be used to directly test some commonly-used hypotheses in modeling granular materials, providing insight into formulating a general theory.

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I arrived in August 2002 with some trepidation, since it was my first time living in the US, and I knew hardly anyone. However, within several weeks I had made many excellent friends through social events at Tang Hall, including **Vikas Anant**, **Julia Cline**, **Natalija Jovanovic**, **Alexandra Kern**, **Karen Lee**, **Vivian Lei**, **Song-**

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