## Chapter 8

## Conclusion

The spot model was initially conceived as a model for particle diffusion, but in this thesis, we have shown that it has a much wider applicability. Coupled with relaxation dynamics, it can be used as the basis of a multiscale model of granular flow. It is a completely new type of model, and it has provided a point of view of granular materials which is unique among the current literature.

In evaluating the successes of the spot simulations, it is perhaps helpful to view them as comprised of two separate ideas: a kinematic description of the mean flow, coupled to the spot model microscopic motion. In terms of the description of the mean flow, the simulations presented here were a good match to DEM and experiment, but it has also been seen that there are some deficiencies. Gaussian velocity profiles which spread diffusively appear only to be approximation to the flow in these situations, and the kinematic description appears to violate some fundamental physical principles (such as frame indifference). However, it should still be viewed as surprising that a model with a single parameter motivated entirely by geometry could work so well, and despite its lack of physics, it provides better answers in some hopper flows than much more complicated models.

Perhaps the largest successes of the spot simulations are in its model of microscopic motion. Regardless of the precise details of the implementation, the concept of local, co-operative motion in dense granular flows appears to have very general validity. Whereas the study of static random packings has been considered by many authors, this work represents the first theoretical model of a flowing random packing, and it has opened up the possibility of asking completely new questions about the nature of amorphous packings. The simulations suggest that in dense granular materials, geometry plays a crucial role, and it seems that any general theory should account for it. One of the failings of traditional plasticity approaches may originate in their continuum formulation, which does not take into account the way in which amorphous packings reorganize. This was one of the underlying principles of the Stochastic Flow Rule, which was explicitly shown in this thesis to provide a good match in two very different geometries.

The notion of strong geometrical constraints has also suggested that correct scale in which to look at physical quantities is the scale of the spot. In the final chapter of this thesis, we showed for the first time the notion of an explicit computational granular element. Using this we were able to directly test continuum hypotheses about granular materials, that would have been difficult to prove in experiment.

## 8.1 Future work

The spot model simulations presented here represent a new simulation technique for granular and amorphous systems, and as such there are a great number of possibilities for developing this work further. The current spot model based on random walks is suited to handle situations in industry which require fast, approximate answers about granular mixing, and one promising direction of research is to investigate ways in which the algorithm could be tuned and optimized. The details of the relaxation step were shown to be unimportant in creating the flowing random packings, and it would be useful to see if this step could be simplified while retaining the majority of the physical results. Possible modifications could be relaxing in a smaller radius, or relaxing only after several spot bulk displacements have occurred.

Other issues of practical importance could also be addressed. In this work, the spot centers were allowed to come within a distance of d of the wall, but tuning this parameter could create more realistic boundary layers. Tuning the biasing of the spot

motion could also affect the development of the avalanching at the free surface. Using ideas from the Stochastic Flow Rule, it should also be reasonably straightforward to create a spot simulation to describe the annular Couette geometry. A careful analysis of the spot motion and its relation to the exponentially decaying velocity profile could yield important physical insight. Adapting the spot model to handle polydisperse packings, or those with irregular shapes, could also be a promising direction. It is possible that the spot picture could show that some of the segregation behavior of polydisperse granular materials [120, 121, 69] has a geometrical origin.

Perhaps the most exciting aspect of the work presented here is in creating a general multiscale model of granular flow. One can envisage a simulation of particles in a granular packing whereby a continuum description of stresses is employed to generate macroscopic flow, but a relaxation model is used to correct geometric packing constraints at a microscopic level. A model of this type could exhibit feedback, whereby information about the amount of particle reordering is coupled back into the macroscopic description, creating a truly multiscale model for granular flow. Since the spot simulations do not require the complex inter-particle forces to be considered, such a model could be several orders of magnitude faster than DEM, providing a practical simulation technique for industry, while also yielding important insight into the basic physics of granular flow.