

# Bibliography

- [1] <http://lammmps.sandia.gov/>.
- [2] <http://web.mit.edu/pebble-bed>.
- [3] <http://www.qhull.org/>.
- [4] See, e.g., <http://gif.inel.gov>, <http://nuclear.inel.gov>.
- [5] <http://www.pbmr.com>.
- [6] M. P. Allen and D. J. Tildesley. *Computer Simulation of Liquids*. Oxford University Press, 1987.
- [7] C. Ancey. *Phys. Rev. E*, 65:011304, 2002.
- [8] I. S. Aranson and L. S. Tsimring. Continuum description of avalanches in granular media. *Phys. Rev. E*, 64:020301, 2001.
- [9] I. S. Aranson and L. S. Tsimring. Continuum theory of partially fluidized granular flows. *Phys. Rev. E*, 65:061303, 2002.
- [10] I. S. Aranson and L. S. Tsimring. Patterns and collective behavior in granular media: Theoretical concepts. *Rev. Mod. Phys.*, 78:641–692, 2006.
- [11] F. Aurenhammer. Voronoi diagrams – a survey of a fundamental geometric data structure. *ACM Computing Surveys*, 23(3):345–405, 1991.
- [12] P. Bak. *How nature works: the science of self-organized criticality*. Copernicus, New York, 1996.
- [13] P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. A*, 38(1):364, 1988.
- [14] C. B. Barber, D. P. Dobkin, and H. T. Huhdaanpaa. The quickhull algorithm for convex hulls. *ACM Transactions on Mathematical Software*, 22(4):469–483, 1996.
- [15] G. W. Baxter and R. P. Behringer. Cellular automata models of granular flow. *Phys. Rev. A*, 42:1017–1020, 1990.
- [16] G. W. Baxter and R. P. Behringer. Cellular automata models for the flow of granular materials. *Physica D*, 52:465–471, 1991.

- [17] M. Z. Bazant. The spot model for random-packing dynamics. *Mechanics of Materials*, 38:717–731, 2006.
- [18] G. Berton, R. Delannay, P. Richard, N. Taberlet, and A. Valance. *Phys. Rev. E*, 68:051303, 2003.
- [19] D. L. Blair, N. Mueggenburg, A. M. Marshall, H. M. Jaeger, and S. R. Nagel. Force distributions in three-dimensional granular assemblies: Effects of packing order and interparticle friction. *Phys. Rev. E*, 63:041304, 2001.
- [20] R. D. Blumofe, C. F. Joerg, B. C. Kuszmaul, C. E. Leiserson, K. H. Randall, and Y. Zhou. Cilk: an efficient multithreaded runtime system. In *PPOPP '95: Proceedings of the fifth ACM SIGPLAN symposium on Principles and practice of parallel programming*, pages 207–216, New York, NY, USA, 1995. ACM Press.
- [21] L. Bocquet, W. Losert, D. Schalk, T. C. Lubensky, and J. P Gollub. *Phys. Rev. E*, 65:011307, 2001.
- [22] D. Bonamy, F. Daviaud, and L. Laurent. *Phys. Rev. Lett.*, 89:034301–1, 2002.
- [23] T. Boutreux and P. G. de Gennes. Compaction of granular mixtures: a free volume model. *Physica A*, 244:59–67, 1997.
- [24] Robert C. Brewster, James W. Landry, Gary S. Grest, and Alex J. Levine. Breakdown of bagnold scaling in cohesive granular flows. *Phys. Rev. E*, 72:061301, 2005.
- [25] H. Caram and D. C. Hong. Random-walk approach to granular flows. *Phys. Rev. Lett.*, 67:828–831, 1991.
- [26] X. Cheng, J. B. Lechman, A. Fernandez-Barbero, G. S. Grest, H. M. Jaeger, G. S. Karczmar, M. E. Mobius, and S. R. Nagel. Three-dimensional shear in granular flow. *Phys. Rev. Lett.*, 96:038001, 2006.
- [27] J. Choi, A. Kudrolli, and M. Z. Bazant. Velocity profile of gravity-driven dense granular flow. *J. Phys.: Condensed Matter*, 17:S2533–S2548, 2005.
- [28] J. Choi, A. Kudrolli, R. R. Rosales, and M. Z. Bazant. Diffusion and mixing in gravity driven dense granular flows. *Phys. Rev. Lett.*, 92:174301, 2004.
- [29] M. H. Cohen and D. Turnbull. Molecular transport in liquids and glasses. *J. Chem. Phys.*, 31:1164, 1959.
- [30] Y. Cohen and A. B. Metzner. Wall effects in laminar flow of fluids through packed beds. *AIChE Journal*, 27:705–714, 1981.
- [31] S. N. Coppersmith, C. h. Liu, S. Majumdar, O. Narayan, and T. A. Witten. Model for force fluctuations in bead packs. *Phys. Rev. E*, 53:4673–4685, 1996.

- [32] P. A. Cundall and O. D. L. Strack. A discrete numerical model for granular assemblies. *Geotechnique*, 29:47, 1979.
- [33] F. da Cruz, S. Emam, M. Prochnow, J. Roux, and F. Chevoir. Rheophysics of dense granular materials: Discrete simulation of plane shear flows. *Phys. Rev. E*, 72:021309, 2005.
- [34] P. G. de Gennes. Granular matter: a tentative view. *Rev. Mod. Phys.*, 71:S374–S382, 1999.
- [35] C. Donati, J. F. Douglas, W. Kob, S. J. Plimpton, P. H. Poole, and S. C. Glotzer. Stringlike cooperative motion in a supercooled liquid. *Phys. Rev. Lett.*, 80:2338–2341, 1998.
- [36] A. Donev, I. Cisse, D. Sachs, E. A. Variano, F. H. Stillinger, R. Connelly, S. Torquato, and P. M. Chaikin. Improving the density of jammed disordered packings using ellipsoids. *Science*, 303:990–993, 2004.
- [37] A. Donev, S. Torquato, F. H. Stillinger, and R. Connelly. Jamming in hard sphere and disk packings. *J. Appl. Phys.*, 95:989–999, 2004.
- [38] C. du Toit. The numerical determination of the variation in the porosity of pebble-bed code. In *Proceedings of the Conference on High-Temperature Reactors, Petten, NL, Vienna, Austria, April 2002*. International Atomic Energy Agency.
- [39] S. F. Edwards. The equations of stress in a granular material. *Physica A*, 249:226–231, 1998.
- [40] T. Elperin and A. Vikhanski. *Europhys. Lett.*, 42:619, 1998.
- [41] D. Ertas and T. C. Halsey. Granular gravitational collapse and chute flow. *Europhys. Lett.*, 60:931–937, 2002.
- [42] H. Eyring. Viscosity, plasticity, and diffusion as examples of absolute reaction rates. *J. Chem. Phys.*, 4:283–291, 1936.
- [43] M. L. Falk and J. S. Langer. Dynamics of viscoplastic deformation in amorphous solids. *Phys. Rev. E*, 57:7192–7205, 1998.
- [44] M. L. Falk, J. S. Langer, and L. Pechenik. Thermal effects in the shear-transition-zone theory of amorphous plasticity: Comparisons to metallic glass data.
- [45] D. Fenistein and M. van Hecke. Wide shear zones in granular bulk flow. *Nature*, 425:256, 2003.
- [46] A. Ferguson and B. Chakraborty. Stress and large-scale spatial structures in dense, driven granular flows. *Phys. Rev. E*, 73(1):011303, 2006.

- [47] A. Ferguson and B. Chakraborty. Spatially heterogenous dynamics in dense, driven granular flows. *Europhysics Letters (EPL)*, 78(2):28003, 2007.
- [48] A. Ferguson, B. Fisher, and B. Chakraborty. Impulse distributions in dense granular flows: Signatures of large-scale spatial structures. *Europhysics Letters*, 66(2):277–283, 2004.
- [49] K. Fukuda, J. Kendall, J. Kupitz, D. Matzner, E. Mulder, P. Pretorius, A. Shenoy, S. Shiozawa, W. Simon, Y. Sun, P. Uselton, and Y. Xu. Current status and future development of modular, high temperature, gas-cooled reactor technology. Technical Report IAEA-TECDOC–1198, International Atomic Energy Agency, Vienna, Austria, 2001.
- [50] J. S. Goodling, R. I. Vachon, W. S. Stelpflug, and S. J. Ying. Radial porosity distribution in cylindrical beds packed with spheres. *Powder Technology*, 35:23–29, 1983.
- [51] H. D. Gougar, W. K. Terry, and A. M. Ougouag. Matrix formulation of pebble circulation in the pebbled code. In *Proceedings of the Conference on High-Temperature Reactors, Petten, NL*, Vienna, Austria, April 2002. International Atomic Energy Agency.
- [52] P. A. Gremaud and J. V. Matthews. On the computation of steady hopper flows: I. stress determination for coulomb materials. *J. Comput. Phys.*, 166:63–83, 2001.
- [53] P. A. Gremaud, J. V. Matthews, and M. O’Malley. On the computation of steady hopper flows: Ii. von mises materials in various geometries. *J. Comput. Phys.*, 200:639–653, 2004.
- [54] P. A. Gremaud, J. V. Matthews, and D. G. Schaeffer. On the computation of steady hopper flows: Iii. model comparisons. *J. Comput. Phys.*, 219:443–454, 2006.
- [55] C. Guáqueta. Computer simulations of a stochastic model for granular drainage, 2003.
- [56] C. h. Liu, S. R. Nagel, D. A. Schecter, N. Coppersmith, S. Majumdar, O. Narayan, and T. A. Witten. Force fluctuations in bead packs. *Science*, 269:513–515, 1995.
- [57] D. W. Howell, R. P. Behringer, and C. T. Veje. Stress fluctuations in a 2D granular couette experiment: A continuous transition. *Phys. Rev. Lett.*, 82(26):5241–5244, 1999.
- [58] S. Hu and R. Wang. Power operation commissioning tests of htr-10. In *Proceedings of the 2nd International Topical Meeting on High Temperature Reactor Technology, Beijing, China*. Institute for Nuclear and New Energy Technology, September 22–24 2004.

- [59] H. M. Jaeger and S. R. Nagel. Physics of the granular state. *Science*, 255:1523–1531, 1992.
- [60] H. M. Jaeger, S. R. Nagel, and R. P. Behringer. Granular solids, liquids, and gases. *Rev. Mod. Phys.*, 68:1259–1273, 1996.
- [61] J. T. Jenkins and S. B. Savage. A theory for the rapid flow of identical, smooth, nearly elastic particles. *J. Fluid Mech.*, 130:187–202, 1983.
- [62] P. Jop, Y. Forterre, and O. Pouliquen. A constitutive law for dense granular flows. *Nature*, 441:727–730, 2006.
- [63] A. Kadak and M. Z. Bazant. Pebble-flow experiments for pebble-bed reactors. In *Proceedings of the 2nd International Topical Meeting on High Temperature Reactor Technology, Beijing, China*. Institute for Nuclear and New Energy Technology, September 22–24 2004.
- [64] L. P. Kadanoff. Built upon sand: Theoretical ideas inspired by the flow of granular materials. *Rev. Mod. Phys.*, 71:435–444, 1999.
- [65] K. Kamrin and M. Z. Bazant. Stochastic flow rule for granular materials. *Phys. Rev. E*, 75:041301, 2007.
- [66] K. Kamrin, C. H. Rycroft, and M. Z. Bazant. The stochastic flow rule: A multi-scale model for granular plasticity. *Modelling. Simul. Mater. Sci. Eng.*, 15:S449–S464, 2007.
- [67] A. R. Kansal, S. Torquato, and F. H. Stillinger. Diversity of order and densities in jammed hard-particle packings. *Phys. Rev. E*, 66:041109, 2002.
- [68] D. V. Khakhar, J. J. McCarthy, T. Shinbrot, and J. M. Ottino. *Phys. Fluids*, 9:31, 1997.
- [69] Z. S. Khan and S. W. Morris. Subdiffusive axial transport of granular materials in a long drum mixer. *Phys. Rev. Lett.*, 94:048002, 2005.
- [70] Z. S. Khan, W. A. Tokaruk, and S. W. Morris. Oscillatory granular segregation in a long drum mixer. *Europhys. Lett.*, 66:212, 2004.
- [71] J. W. Landry, G. S. Grest, L. E. Silbert, and S. J. Plimpton. Confined granular packings: structure, stress, and forces. *Phys. Rev. E*, 67:041303, 2003.
- [72] M. Latzel, S. Luding, H. J. Herrmann, D. W. Howell, and R.P. Behringer. Comparing simulation and experiment of a 2D granular couette shear device. *Euro. Phys. Journ. E.*, 11:325–333, 2003.
- [73] A. Lemaître. A dynamical approach to glassy materials. 2002.
- [74] A. Lemaître. Origin of a repose angle: Kinetics of rearrangements for granular materials. *Phys. Rev. Lett.*, 89:064303, 2002.

- [75] A. Lemaître. Rearrangements and dilatency for sheared dense materials. *Phys. Rev. Lett.*, 89:195503, 2002.
- [76] J. Litwiniszyn. Statistical methods in the mechanics of granular bodies. *Rheol. Acta*, 2/3:146, 1958.
- [77] J. Litwiniszyn. *Bull. Acad. Pol. Sci.*, 9:61, 1963.
- [78] J. Litwiniszyn. The model of a random walk of particles adapted to researches on problems of mechanics of loose media. *Bull. Acad. Pol. Sci.*, 11:593, 1963.
- [79] A. J. Liu and S. R. Nagel. Jamming is not just cool any more. *Nature (London)*, 396:21, 1998.
- [80] W. Losert, L. Bocquet, T. C. Lubensky, and J. P. Gollub. Particle dynamics in sheared granular matter. *Phys. Rev. Lett.*, 85:1428–1431, 2000.
- [81] B. D. Lubachevsky and F. H. Stillinger. Geometric properties of random disk packings. *J. Stat. Phys.*, 60:561–583, 1990.
- [82] T. S. Majumdar and R. P. Behringer. Contact force measurements and stress-induced anisotropy in granular materials. *Nature*, 435:1079–1082, 2005.
- [83] A. Medina, J. Andrade, and C. Trevino. Experimental study of the tracer in the granular flow of a 2D silo. *Physics Letters A*, 249:63–68, 1998.
- [84] A. Medina, J. A. Cordova, E. Luna, and C. Trevino. Velocity field measurements in granular gravity flow in a near 2D silo. *Physics Letters A*, 220:111–116, 1998.
- [85] M. Menon and D. J. Durian. Diffusing-wave spectroscopy of dynamics in a three-dimensional granular flow. *Science*, 275:1920–1922, 1997.
- [86] E. A. Merritt and D. J. Bacon. Raster3D: Photorealistic molecular graphics. *Meth. Enzymol.*, 277:505–524, 1997.
- [87] G. D. R. Midi. On dense granular flows. *Euro. Phys. Journ. E.*, 14:341–365, 2004.
- [88] D. E. Mueth, G. F. Debregeas, G. S. Karczmar, P. J. Eng, S. R. Nagel, and H. M. Jaeger. Signatures of granular microstructure in dense shear flows. *Nature*, 406:385–388, 2000.
- [89] D. M. Mueth, H. M. Jaeger, and S. R. Nagel. Force distribution in a granular medium. *Phys. Rev. E*, 57:3164–3169, 1998.
- [90] J. Mullins. Stochastic theory of particle flow under gravity. *J. Appl. Phys.*, 43:665, 1972.
- [91] J. Mullins. Experimental evidence for the stochastic theory of particle flow under gravity. *Powder Technology*, 9:29, 1974.

- [92] J. Mullins. Critique and comparison of two stochastic theories of gravity-induced particle flow. *Powder Technology*, 23:115–119, 1979.
- [93] V. V. R. Natarajan, M. L. Hunt, and E. D. Taylor. *J. Fluid Mech.*, 304:1, 1995.
- [94] R. M. Nedderman. *Statics and Kinematics of Granular Materials*. Nova Science, 1991.
- [95] R. M. Nedderman and U. Tüzün. Kinematic model for the flow of granular materials. *Powder Technology*, 22:243, 1979.
- [96] M. Newey, J. Ozik, S. M. van der Meer, E. Ott, and W. Losert. Band-in-band segregation of multidisperse granular mixtures. *Europhys. Lett.*, 66:205, 2004.
- [97] D. Nicholls. The pebble-bed modular reactor. *Nuclear News*, 44, 2001.
- [98] C. S. O’Hern, S. A. Langer, A. J. Liu, and S. R. Nagel. *Phys. Rev. Lett.*, 88:075507, 2002.
- [99] C. S. O’Hern, L. E. Silbert, A. J. Liu, and S. R. Nagel. Jamming at zero temperature and zero applied stress: The epitome of disorder. *Phys. Rev. E*, 68:011306, 2003.
- [100] G. Y. Onoda and E. G. Liniger. Random loose packing of uniform spheres and the dilatancy onset. *Phys. Rev. Lett.*, 64:2727, 1990.
- [101] A. V. Orpe and D. V. Khakhar, 2001.
- [102] A. V. Orpe and A. Kudrolli. Velocity correlations in dense granular flows observed with internal imaging. *Phys. Rev. Lett.*, 98:238001, 2007.
- [103] A. M. Ouguag, J. J. Cogliati, and J.-L. Kloosterman. Methods for modeling the packing of fuel elements in pebble-bed reactors. In *Proceedings of the Topical Meeting on Mathematics and Computation, Avignon, France*, LaGrange Park, IL, September 2005. American Nuclear Society.
- [104] J. Palacci. Computer simulations of granular materials with periodic boundary conditions using the spot model. Technical report, MIT Department of Mathematics, 2005.
- [105] S. Patankar. *Numerical Heat Transfer and Fluid Flow*. Taylor & Francis, 1980.
- [106] E. B. Pitman and D. G. Schaeffer. Stability of time dependent compressible granular flow in two dimensions. *Commun. Pure. Appl. Math.*, 40:421–447, 1987.
- [107] O. Pouliquen. *Phys. Fluids*, 11:542, 1999.
- [108] J. R. Prakash and K. K. Rao. *J. Fluid Mech.*, 225:21, 1991.

- [109] J. Rajchenbach. *Phys. Rev. Lett.*, 65:2221, 1990.
- [110] S. Reiss. Let a thousand reactors bloom. *Wired Magazine*, 12.09, 2004.
- [111] H. Risken. *The Fokker-Planck Equation*. Springer, 1996.
- [112] C. H. Rycroft, M. Z. Bazant, G. S. Grest, and J. W. Landry. Dynamics of random packings in granular flow. *Phys. Rev. E*, 73:051306, 2006.
- [113] C. H. Rycroft, G. S. Grest, J. W. Landry, and M. Z. Bazant. Analysis of granular flow in a pebble-bed nuclear reactor. *Phys. Rev. E*, 74:021306, 2006.
- [114] A. Samadani, A. Pradhan, and A. Kudrolli. Size segregation of granular matter in silo drainage. *Phys. Rev. E*, 60:7203–7209, 1999.
- [115] S. B. Savage. Gravity flow of cohesionless granular materials in chutes and channels. *J. Fluid Mech.*, 92:53–96, 1979.
- [116] S. B. Savage. *J. Fluid Mech.*, 377:1, 1998.
- [117] D. G. Schaeffer. Instability in the evolution equations describing incompressible granular flow. *J. Diff. Eq.*, 66:19–50, 1987.
- [118] A. Schofield and C. Wroth. *Critical State Soil Mechanics*. McGraw-Hill, 1968.
- [119] A. J. Sederman, P. Alexander, and L. F. Gladden. Structure of packed beds probed by magnetic resonance imaging. *Powder Technology*, 117:255–269, 2001.
- [120] T. Shinbrot. The brazil nut effect – in reverse. *Nature*, 429:352–353, 2004.
- [121] T. Shinbrot and F. J. Muzzio. Reverse buoyancy in shaken granular beds. *Phys. Rev. Lett.*, 81:4365, 1998.
- [122] S. Siavoshi, A. V. Orpe, and A. Kudrolli. *Phys. Rev. E*, 73:010301(R), 2006.
- [123] L. E. Silbert, D. Ertas, G. S. Grest, T. C. Halsey, D. Levine, and S. J. Plimpton. Granular flow down an inclined plane: Bagnold scaling and rheology. *Phys. Rev. E*, 64(5):051302, Oct 2001.
- [124] L. E. Silbert, D. Ertas, G. S. Grest, T. C. Halsey, and D. Levine. Geometry of frictionless and frictional sphere packings. *Phys. Rev. E*, 65:031304, 2002.
- [125] L. E. Silbert, G. S. Grest, and J. W. Landry. Statics of the contact network in frictional and frictionless granular packings. *Phys. Rev. E*, 66:061303, 2002.
- [126] L. E. Silbert, J. W. Landry, and G. S. Grest. Granular flow down a rough inclined plane: transition between thin and thick piles. *Phys. Fluids*, 15:1, 2003.
- [127] V. V. Sokolovskii. *Statics of Granular Materials*. Pergamon/Oxford, 1965.



- [128] F. Spaepen. A microscopic mechanism for steady state inhomogeneous flow in metallic glasses. *Acta Metallurgica*, 25:407–415, 1977.
- [129] J. Sun, F. Battaglia, and S. Subramaniam. Dynamics and structures of segregation in a dense, vibrating granular bed. *Phys. Rev. E*, 74:061307, 2006.
- [130] David Talbot. The next nuclear power plant. *MIT Technology Review*, pages 54–59, Jan/Feb 2002.
- [131] W. K. Terry, H. D. Gougar, and A. M. Ougouag. Direct deterministic method for neutronics analysis and computation of asymptotic burnup distribution in a recirculating pebble-bed reactor. *Annals of Nuclear Energy*, 29:1345–1364, 2002.
- [132] S. Torquato. *Random Heterogeneous Materials*. Springer, New York, 2003.
- [133] S. Torquato and F. H. Stillinger. Local density fluctuations, hyperuniformity, and order metrics. *Phys. Rev. E*, 68:041113, 2003.
- [134] S. Torquato, T. M. Truskett, and P. G. Debenedetti. Is random close packing of spheres well defined? *Phys. Rev. Lett.*, 84:2064, 2000.
- [135] J.-C. Tsai and J. P. Gollub. Slowly sheared dense granular flows: Crystallization and nonunique final states. *Phys. Rev. E*, 70:031303, 2004.
- [136] J.-C. Tsai, G. A. Voth, and J. P. Gollub. Internal granular dynamics, shear-induced crystallization, and compaction steps. *Phys. Rev. Lett.*, 91:064301, 2003.
- [137] U. Tüzün and R. M. Nedderman. Experimental evidence supporting the kinematic modelling of the flow of granular media in the absence of air drag. *Powder Technology*, 23:257, 1979.
- [138] B. Utter and R. P. Behringer. Transients in sheared granular matter. *Eur. Phys. J. E*, 14:373–380, 2004.
- [139] C. T. Veje, D. W. Howell, and R. P. Behringer. Kinematics of a two-dimensional granular couette experiment at the transition to shearing. *Phys. Rev. E*, 59:739–745, 1999.
- [140] G. Voronoi. Nouvelles applications des paramètres continus à la théorie des formes quadratiques. *Journal für die Reine und Angewandte Mathematik*, 133:97–198, 1908.
- [141] D. Vortmeyer and J. Schuster. Evaluation of steady flow profiles in rectangular and circular packed beds by a variational method. *Chemical Engineering Science*, 38:1691–1699, 1983.

- [142] E. R. Weeks, J. C. Crocker, A. C. Levitt, A. Schofield, and D. A. Weitz. Three-dimensional direct imaging of structural relaxation near the colloidal glass transition. *Science*, 287:627–631, 2000.
- [143] S. M. White and C. L. Tien. Analysis of flow channelling near the wall in packed beds. *Wärme- und Stoffübertragung*, 21:291–296, 1987.
- [144] H. P. Zhu and A. B. Yu. Steady-state granular flow in a three-dimensional cylindrical hopper with flat bottom: microscopic analysis. *J. Phys. D*, 37:1497, 2004.