Math 126: Homework 12

1. Carry out Exercise 4.6 from the textbook on Burgers' equation. In addition, find an example of an unphysical integral solution to the exercise, where the entropy condition is not satisfied.

2. (a) Consider the traffic flow equation $\rho_t + v_m(1 - \frac{2\rho}{\rho_m})\rho_x = 0$ with initial condition

$$\rho(x,0) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{\rho_m}{2} & \text{for } x \geq 0. \end{cases}$$

Solve the equation by making use of the Rankine–Hugoniot condition to determine the shock velocity.

(b) Consider the traffic flow equation with a small viscous damping term added:

$$\rho_t + v_m \left(1 - \frac{2\rho}{\rho_m}\right) \rho_x = \epsilon \rho_{xx}.$$

Find a travelling solution of the form $\rho(x,t) = U(x - vt) = U(\xi)$ such that

$$\lim_{\xi \to -\infty} U(\xi) = 0, \qquad \lim_{\xi \to +\infty} U(\xi) = \frac{\rho_m}{2}.$$

Sketch the solution and show that it approaches the solution from part (a) as $\epsilon \to 0$. *[Hint: to find the solution, it is possible to follow the general treatment described in Section 4.4.6 of the textbook.]*