Math 221 – Practice problems for the first midterm (fall 2012, lecture 1)

1. If f is a function with $f(2-3x)=\sin(\pi x^2)$ for all x, then compute $f(0)$; also compute $f(x)$ for arbitrary x.

2. If f is a function with $f(x)=\dfrac{4}{2+x}$ for all $x\neq -2$, then find $f^{-1}(-2)$; also find $f^{-1}(x)$ for arbitrary x.

3. Use the $\epsilon-\delta$ definition of limit to show that $\lim_{x\to 1}\frac{x-1}{x+1}=0$.

4. Same question as before for the limit $\lim_{x\to -1} x^2+2x-1=-2$. Be sure to include a concluding statement.

5. Use the limit properties to compute $\displaystyle\lim_{x\to 3}\frac{x^2-2x-3}{x^3-9x}$; show **all** the steps.

6. *True or False?* If $\displaystyle\lim_{x\to 0} f(x)=0$ then $\displaystyle\lim_{x\to 0}\frac{f(x)}{x}$ exists. **Explain your answer.**

►►► See also problems 19, 20, 21 on page 51 of the text.

7. Find ***all*** the asymptotes of the graph of the function $y=f(x)=\dfrac{x^2+2x}{x+1}$.

8. Show that the line $y=x+1$ is an asymptote of the graph of $y=\sqrt{x^2+2x}$ as $x\to\infty$.

9. Find the vertical asymptotes of the graph of $y=\dfrac{x^2-5x+4}{x^2-1}$

10. Find the horizontal asymptotes of $y=\dfrac{x+1}{\sqrt{x^2+4}}$.

11. Compute the limits

$$\lim_{x\to\pi}\frac{\sin x}{2\pi-2x} \qquad \lim_{x\to 0}\frac{\cos(bx)-1-x}{\sin(ax)} \qquad \lim_{x\to\pi/2}\frac{1-\sin x}{\pi-2x}$$

$$\lim_{x\to\infty}\frac{\cos ax}{x+a} \qquad \lim_{x\to\infty}\frac{x\sin x}{x^2+1} \qquad \lim_{x\to\pi/2}\frac{1-\sin x}{(\pi-2x)^2}$$

$$\lim_{x\to+\infty}\frac{x^2+4x}{(x+2)(2x-3)} \qquad \lim_{x\to\frac{\pi}{2}}(2x-\pi)\tan x \qquad \lim_{x\to 0}\frac{\cos(ax)-1}{\tan(bx^2)}$$

Here a and b are positive constants.

12. The function $f(x)$ is continuous on $(-\infty,+\infty)$ and

$$f(x)=\frac{\sin(x^2)}{x^2}$$

for all $x\neq 0$. Find $f(0)$.

13. If the function $f(x)=\begin{cases} x^2+ax & (x>2)\\ \frac{4}{x} & (x\leq 2)\end{cases}$ is continuous at $x=2$, then what is a?

14. Is the function $f(x)=|x|+x^2$ differentiable at $x=0$? Use the definition of the derivative to justify your answer.

15. Consider the function $f(x)$ defined by $f(x)=\begin{cases}\sin(kx) & \text{if } x\leq 0\\ 3x & \text{if } x>0\end{cases}$ where k is a constant and answer the following questions.

(1) For what value (or values) of k is $f(x)$ continuous at $x=0$? Justify your answer.
(2) For what value (or values) of k is $f(x)$ differentiable at $x=0$? Justify your answer.

16. Use **the definition of the derivative as a limit** (you MUST use this) to find the derivative of $f(x) = \sqrt{x^2+3}$ at $x = 1$ and the **equation of the tangent line** to the graph of $f(x)$ at $x = 1$.

17. Compute the derivative of each of these functions

$$y = x^2 + ax + b, \qquad y = \frac{x^3+x}{x^2+1} \qquad y = \frac{(3+2x)^2}{3+2x^2}$$

$$y = (x+a)^2, \qquad y = a(x^2-3)^4 \qquad y = \frac{a}{x+b}$$

$$y = \frac{x^4+x}{x^2+2} \qquad y = (x^2-x)^5 \qquad y = \frac{ax+b}{cx+d}$$

$$y = \left(\cos(x^2) + x\cos^2 x\right)^4, \qquad y = \tan(\sqrt{x^2+1}), \qquad y = \frac{\sin(x^2+1)}{\cos(x-1)}.$$

Here a, b, c, d are positive constants.

18. Find the slope of the tangent to the graph of $y = \left(x^2-1\right)^2$ at the point on the graph where $x = 1$.

19. (prof. Thieffault, fall 2010) An airplane on a runway is taking off. Its distance from the starting point is $x(t) = \frac{10}{9}t^2$ (distance x in meters; time t is measured in seconds).

- What is the plane's velocity at time t?
- What is the plane's velocity at time $t = 0$?
- The plane becomes airborne when its velocity is 60 $^{\mathrm{m}}/_{\mathrm{sec}}$. At what time t does the plane have this velocity?
- How far will the plane have gone when it has that velocity?

20. For each of the problems, circle the (unique) correct answer.

(1) Only one of the following limits exists. Which one?

A. $\lim_{x\to\infty} \cos(\frac{1}{x})$ B. $\lim_{x\to\infty} \cos x$ C. $\lim_{x\to 0} \cos(\frac{1}{x})$ D. $\lim_{x\to 0} \frac{\cos x}{x}$

(2) One of the following functions is continuous but not differentiable at $x = 0$. Which one?

A. $f(x) = x|x|$ B. $f(x) = \sin(\frac{1}{x})$

C. $f(x) = \begin{cases} x\sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$ D. $f(x) = \begin{cases} x^2\sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$

(3) Only one of the following statements is correct. Which one?

A. If $\lim_{x\to a} f(x) = 1$, then $f(a)$ must be 1.

B. The 11th order derivative of $\sin x$ is $\cos x$.

C. If $f(x)$ and $g(x)$ are functions defined near $x = 1$ and $\lim_{x\to 1} g(x) = 0$, then $\lim_{x\to 1} \frac{f(x)}{g(x)}$ does not exist.

D. If $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists, then $\lim_{x\to a} f(x) = f(a)$.

Answers and Hints

(1) Set $u = 2 - 3x$. Then $x = (2-u)/3$, and we get $f(u) = \sin\bigl(\pi\bigl(\frac{2-u}{3}\bigr)^2\bigr)$. This is true for any value of u. Therefore $f(x)$ is given by $f(x) = \sin\bigl(\pi\bigl(\dfrac{2-x}{3}\bigr)^2\bigr)$

(2) By definition $y = f^{-1}(x)$ if $x = f(y)$, i.e. in this problem if $x = \dfrac{4}{2+y}$. Solve this equation for y to get $y = \dfrac{4}{x} - 2$. Hence $f^{-1}(-2) = \dfrac{4}{-2} - 2 = -4$.

(3)

$|f(x) - L| = |\dfrac{x-1}{x+1} - 0| = |\dfrac{1}{x+1}||x-1|$ we need to bound $|\dfrac{1}{x+1}|$, so assume $\delta = 1$

$0 < x < 2 \;=>\; 1 < x+1 < 3 \;=>\; 1 > \dfrac{1}{x+1} > \dfrac{1}{3}$ and since $|x+1| = x+1$

$|\dfrac{1}{x+1}||x-1| < |x-1| < \delta$ from here it suffices to choose $\delta = \epsilon$

We need to have both properties $|x-1| < \epsilon$ and $\delta = 1$ (that is, x at a distance less than 1 and at a distance less than ϵ from 1), so x at a distance δ less or equal to the smallest of the two values (1 and ϵ) will satisfy both properties. That is: given any ϵ, choosing $\delta = \text{smallest}\{1, \epsilon\}$ will guaranteed that whenever $|x-1| < \delta$, $|\frac{x-1}{x+1} - 0| < \epsilon$. (Note: you could have just as well said that if $0 < x < 2$ then $\frac{1}{x+1} < 1$ since x is positive and in the denominator. But the method above always works.)

(4)

$|f(x) - L| = |x^2 + 2x - 1 + 2| = |x^2 + 2x + 1| = |x+1|^2 < \delta^2$

from here it suffices to choose $\delta = \sqrt{\epsilon}$ to guarantee that if $|x - (-1)| < \delta$ then $|f(x) - (-2)| < \epsilon$

(5)

$$
\begin{aligned}
\lim_{x\to 3} \frac{x^2-2x-3}{x^3-9x} &= \lim_{x\to 3} \frac{(x-3)(x+1)}{x(x^2-9)} && \text{factor} \\
&= \lim_{x\to 3} \frac{(x-3)(x+1)}{x(x-3)(x+3)} && \text{factor some more} \\
&= \lim_{x\to 3} \frac{x+1}{x(x+3)} && \text{cancel common factors} \\
&= \frac{\lim_{x\to 3}(x+1)}{\lim_{x\to 3} x(x+3)} && \text{limit property for quotients} \\
&= \frac{\lim_{x\to 3} x + \lim_{x\to 3} 1}{\lim_{x\to 3} x \cdot \lim_{x\to 3}(x+3)} && \begin{array}{r}\text{limit of sum (or product) is} \\ \text{sum (or product) of limits}\end{array} \\
&= \frac{3+1}{3\big((\lim_{x\to 3} x)+3\big)} \\
&= \frac{4}{3\cdot(3+3)} \\
&= \frac{2}{9}.
\end{aligned}
$$

(6) This is not true. To back our claim that the statement is not true we have to find one counterexample. We only need one counterexample, but it turns out there are many, so different people are likely to come up with different counterexamples. One possible counterexample is $f(x) = |x|$. For this function $\lim_{x\to 0} |x| = 0$, while $\lim_{x\to 0} \dfrac{f(x)}{x} = \lim_{x\to 0} \dfrac{|x|}{x}$ does not exist since the left and right hand limits are not equal.

(7) **Vertical asymptotes** appear at points where $\lim_{x\to a} f(x)$ does not exist. For this function that only happens when $x = -1$. If $x > -1$ is very close to -1 then we have

$$\frac{x^2+2x}{x+1} \approx \frac{-1}{x+1}$$

Here $x+1$ is a very small positive number, so $-1/(x+1)$ will be a large negative number. Therefore

$$\lim_{x\searrow -1} \frac{x^2+2x}{x+1} = -\infty.$$

Similarly,

$$\lim_{x\nearrow -1} \frac{x^2+2x}{x+1} = +\infty.$$

Thus the line $x = -1$ is a vertical asymptote and the graph approaches both ends of the line.

Slanted asymptotes are lines with equation $y = mx + n$. If such an asymptote exists then $m = \lim_{x\to\infty} f(x)/x$ and $n = \lim_{x\to\infty} f(x) - mx$. If both these limits exist then $y = mx + n$ is indeed an asymptote of the graph of $y = f(x)$ as $x \to +\infty$. You can also look for asymptotes in the other direction. You find these by taking the limits for $x \to -\infty$ instead of $+\infty$. By looking for slanted asymptotes you also automatically find **horizontal asymptotes** since these are just slanted asymptotes with $m = 0$.

We find

$$m = \lim_{x\to\infty} \frac{x^2+2x}{x(x+1)} = \lim_{x\to\infty} \frac{x^2}{x^2}\,\frac{1+2/x}{1\cdot(1+1/x)} = 1.$$

So **IF** there is a slanted asymptote at $x\to\infty$ its slope must be $m=+1$. Next we must compute n:

$$\begin{aligned}
n &= \lim_{x\to\infty} \frac{x^2+2x}{x+1} - \overset{(m)}{1}\cdot x \\
&= \lim_{x\to\infty} \frac{x^2+2x-x(x+1)}{x+1} \\
&= \lim_{x\to\infty} \frac{x^2+2x-x^2-x}{x+1} \\
&= \lim_{x\to\infty} \frac{x}{x+1} \\
&= 1.
\end{aligned}$$

Since this limit exists we know that the graph of our function has a slanted asymptote as $x\to\infty$ and that it is given by $y = mx+n = x = 1$.

If you compute the same limits for $x\to-\infty$ you will find that the graph of f also has slanted asymptotes at $x\to-\infty$, and that it is given by the same line, $y=x+1$.

Since any horizontal asymptote would also be a slanted asymptote, and since the slanted asymptote we found has slope $m = 1 \neq 0$, the graph of this function does not have a horizontal asymptote.

(8) First we can try to find the slope of the line, m. For this we calculate the limit $\lim_{x\to\infty} \frac{f(x)}{x}$ which, if it exist, will be the slope.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2x}}{x} = \lim_{x\to\infty} \frac{x}{x}\sqrt{1+\frac{2}{x}} = 1 = m.$$

We then try to find n by calculating the limit

$$\begin{aligned}
\lim_{x\to\infty} (f(x)-mx) &= \lim_{x\to\infty} \sqrt{x^2+2x} - x \\
&= \lim_{x\to\infty} \frac{(\sqrt{x^2+2x}-x)(\sqrt{x^2+2x}+x)}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{2x}{x}\,\frac{1}{\sqrt{1+\frac{2}{x}}+1} = 1 = n.
\end{aligned}$$

The slanted asymptote is $y = mx+n = x+1$.

(9) The function can only have vertical asymptotes at the points where the denominator vanishes, i.e. at $x=\pm1$. To see if $x=-1$ is a vertical asymptote we compute

$$\lim_{x\to-1} f(x) = \lim_{x\to-1} \frac{x^2-5x+4}{x^2-1} = \lim_{x\to-1} \frac{(x-4)(x-1)}{(x+1)(x-1)} = \lim_{x\to-1} \frac{x-4}{x+1} \text{ which does not exist.}$$

In fact, the limit is $+\infty$ or $-\infty$ depending on the direction in which x approaches -1. We conclude that $x=-1$ is a vertical asymptote for the function.

To see if there is a vertical asymptote at $x = +1$ we compute

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2 - 5x + 4}{x^2 - 1} = \lim_{x\to 1} \frac{x-4}{x+1} = \frac{1-4}{1+1} = -\frac{3}{2}.$$

Since the limit exists there is no asymptote at $x = +1$.

(10) To find horizontal asymptotes compute the limits $\lim_{x\to\pm\infty} f(x)$. They are:

$$\begin{aligned}
\lim_{x\to\infty} \frac{x+1}{\sqrt{x^2+4}} &= \lim_{x\to\infty} \frac{x}{\sqrt{x^2}} \frac{1+1/x}{\sqrt{1+4/x^2}} && \text{factor highest power of } x \text{ out of top\&bottom} \\
&= \lim_{x\to\infty} \frac{x}{x} \frac{1+1/x}{\sqrt{1+4/x^2}} && x > 0 \implies \sqrt{x^2} = x \\
&= 1,
\end{aligned}$$

and

$$\begin{aligned}
\lim_{x\to-\infty} \frac{x+1}{\sqrt{x^2+4}} &= \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}} \frac{1+1/x}{\sqrt{1+4/x^2}} && \text{factor highest power of } x \text{ out of top\&bottom} \\
&= \lim_{x\to-\infty} \frac{x}{-x} \frac{1+1/x}{\sqrt{1+4/x^2}} && x < 0 \implies \sqrt{x^2} = -x \\
&= -1.
\end{aligned}$$

Therefore the line $y = +1$ is a horizontal asymptote for $x \to +\infty$, and the line $y = -1$ is a horizontal asymptote for $x \to -\infty$.

To see a graph of the function with the asymptotes type "`y=(x+1)/sqrt(x^2+4),y=1,y=-1`" into Google. Then zoom out horizontally to see the horizontal asymptotes.

(11)

- $\displaystyle\lim_{x\to\pi} \frac{\sin x}{2\pi - 2x} = +\frac{1}{2}$ (substitute $\theta = \pi - x$.)

$$\lim_{x\to 0} \frac{\cos(bx) - 1 - x}{\sin(ax)} = \lim_{x\to 0} \frac{\cos(bx) - 1 - x}{ax} \frac{ax}{\sin(ax)} = \lim_{x\to 0} \frac{\cos(bx) - 1 - x}{ax} \lim_{x\to 0} \frac{ax}{\sin(ax)}$$

The last limit is $\lim_{x\to 0} \frac{ax}{\sin(ax)} = 1$, which you find by substituting $\theta = ax$. The other limit is

$$\begin{aligned}
\lim_{x\to 0} \frac{\cos bx - 1}{ax} - \frac{x}{ax} &= \lim_{x\to 0} \frac{\cos bx - 1}{ax} - \frac{1}{a} \\
&= \lim_{\theta\to 0} \frac{\cos\theta - 1}{\frac{a}{b}\theta} - \frac{1}{a} && \text{substitute } \theta = bx \\
&= -\frac{1}{a}.
\end{aligned}$$

- $\displaystyle\lim_{x\to\pi/2} \frac{1 - \sin x}{\pi - 2x} = \lim_{\theta\to 0} \frac{1 - \cos\theta}{2\theta} = 0.$ Substitute $\theta = \frac{\pi}{2} - x$.

- $\displaystyle\lim_{x\to\pi/2} \frac{1 - \sin x}{(\pi - 2x)^2} = \lim_{\theta\to 0} \frac{1 - \cos\theta}{(2\theta)^2} = \frac{1}{8}$ Same substitution.

- $\displaystyle\lim_{x\to\infty} \frac{\cos ax}{x + a}$

Use the Sandwich Theorem! Since $-1 \le \cos(\text{whatever}) \le +1$ and since $x + a > 0$ when $x \to \infty$, we have

$$\frac{-1}{x+a} \le \frac{\cos ax}{x+a} \le \frac{+1}{x+a}$$

We also have $\lim_{x\to\infty} \dfrac{\pm 1}{x+a} = 0$. The Sandwich Theorem therefore implies that

$$\lim_{x\to\infty} \frac{\cos ax}{x+a} = 0.$$

- $\displaystyle\lim_{x\to\infty} \frac{x \sin x}{x^2+1} = 0$. Again use the Sandwich Theorem, and $\displaystyle\lim_{x\to\infty} \frac{x}{x^2+1} = \lim_{x\to\infty} \frac{x}{x^2} \frac{1}{1+(1/x)^2} = 0.$

- $\displaystyle\lim_{x\to+\infty} \frac{x^2+4x}{(x+2)(2x-3)} = \lim_{x\to\infty} \frac{x^2}{x^2} \frac{1+\frac{4}{x}}{(1+\frac{2}{x})(2-\frac{3}{x})} = \frac{1}{2}.$

- $\displaystyle\lim_{x\to\pi/2} (2x-\pi)\tan x = \lim_{x\to\pi/2} (2x-\pi)\frac{\sin x}{\cos x} = \lim_{x\to\pi/2} (2x-\pi)\frac{\cos(\pi/2-x)}{\sin(\pi/2-x)}.$

Now substitute $u = \pi/2 - x$. If $x \to \pi/2$, then $u \to 0$, and therefore

$$\lim_{x\to\pi/2} (2x-\pi)\tan x = \lim_{u\to 0} (-2u)\frac{\cos u}{\sin u} = -2 \lim_{u\to 0} \cos u \frac{u}{\sin u} = -2.$$

- $\displaystyle\lim_{x\to 0} \frac{\cos(ax)-1}{\tan(bx^2)} = \lim_{x\to 0} \frac{(\cos(ax)-1)(\cos(bx^2))}{\sin(bx^2)}.$

We now look for limits we are familiar with, that is, $\frac{1-\cos(ax)}{(ax)^2}$ and $\frac{\sin(bx^2)}{bx^2}$. We get

$$\lim_{x\to 0} \frac{(\cos(ax)-1)(\cos(bx^2))}{\sin(bx^2)} = \lim_{x\to 0} \cos(bx^2)\frac{(\cos(ax)-1)}{(ax)^2} \frac{bx^2}{\sin(bx^2)} \frac{(ax)^2}{bx^2} = -\frac{1}{2}\frac{a^2}{b}.$$

(12) If the function is continuous, $f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin(x^2)}{x^2}$ so we need to find that limit

$$\lim_{x\to 0} \frac{\sin(x^2)}{x^2} = \big(\text{we make } u = x^2 \text{ and if } x \to 0 \text{ then } u \to 0 \text{ also}\big)$$

$$= \lim_{u\to 0} \frac{\sin u}{u} = 1.$$

Therefore, $f(0) = 1$.

(13) The function is continuous at $x = 2$ if $\lim_{x\to 2} f(x) = f(2)$. By definition $f(2) = 4/2 = 2$. To find the limit we compute left and right limits:

$$\lim_{x\nearrow 2} f(x) = \lim_{x\nearrow 2} \frac{4}{x} = \frac{4}{2} = 2 \text{ and } \lim_{x\searrow 2} f(x) = \lim_{x\searrow 2} x^2 + ax = 4 + 2a.$$

The limit does not exist unless these two limits are equal. So $\lim_{x\to 2} f(x)$ exists only when $2 = 4 + 2a$, i.e. only when $a = -1$.

When $a = -1$ the left and right limits both equal 2, and hence $\lim_{x\to 2} f(x) = 2$. Since $f(2)$ is also 2 we see that $f(x)$ is continuous at $x = 2$ when $a = -1$.

(14) The function $f(x) = |x| + x^2$ is differentiable at $x = 0$ if the limit

$$f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{|x| + x^2}{x} = \lim_{x\to 0} \frac{|x|}{x} + x$$

exists. The left and right limits are

$$\lim_{x\nearrow 0} \frac{|x|}{x} + x = \lim_{x\nearrow 0} -1 + x = -1 \qquad \text{and} \qquad \lim_{x\searrow 0} \frac{|x|}{x} + x = \lim_{x\searrow 0} 1 + x = +1$$

These are not the same, so the limit does not exist, and therefore the function is not differentiable at $x = 0$.

(15) The function is continuous everywhere except for perhaps the point $x = 0$, so to make the function continuous we take side limits at 0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sin(kx) = 0; \qquad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3x = 0$$

so the function is always continuous, for any k. We now look into whether or not the slopes on the right and on the left are equal also. The slope on the left is the derivative of $\sin(kx)$, that is $\cos(kx)k$ evaluated at $x = 0$, the left slope is k. The slope on the right is the derivative of $3x$, that is the right slope is 3. Therefore, the function is differentiable whenever $k = 3$.

(16) We write the definition of derivative as a limit

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{(1+h)^2+3} - 2}{h} = \lim_{h\to 0} \frac{(\sqrt{(1+h)^2+3} - 2)(\sqrt{(1+h)^2+3} + 2)}{h(\sqrt{(1+h)^2+3} + 2)}$$

$$= \lim_{h\to 0} \frac{h^2 + 2h}{h(\sqrt{(1+h)^2+3} + 2)} = \lim_{h\to 0} \frac{h + 2}{\sqrt{(1+h)^2+3} + 2} = 1/2.$$

The tangent line of the function has slope $m = 2$ and goes through the point $(1, f(1)) = (1, 2)$. The equation is $y = 2x$.

(17)

$$\frac{dx^2 + ax + b}{dx} = 2x + a.$$

$$\frac{d\dfrac{x^3+x}{x^2+1}}{dx} = \frac{(3x^2+1)(x^2+1) - (x^3+x)(2x)}{\left(x^2+1\right)^2} = (\text{yada})^2$$

$$\frac{d\dfrac{(3+2x)^2}{3+2x^2}}{dx} = \frac{2(3+2x)\cdot 2\cdot (3+2x^2) - (3+2x)^2\cdot 4x}{\left(3+2x^2\right)^2} = \cdots$$

$$\frac{d(x+a)^2}{dx} = 2(x+a)$$

$$\frac{da(x^2-3)^4}{dx} = a\frac{d(x^2-3)^4}{dx} = a\; 4(x^2-3)^3\frac{d(x^2-3)}{dx} = 8ax(x^2-3)^3$$

$$\frac{d\dfrac{a}{x+b}}{dx} = \frac{(x+b)\frac{da}{dx} - a\frac{d(x+b)}{dx}}{(x+b)^2} = \frac{-a}{(x+b)^2}$$

$$\frac{d\dfrac{x^4+x}{x^2+2}}{dx} = \frac{(4x^3+1)(x^2+2)-(x^4+x)2x}{(x^2+2)^2} = \cdots$$
$$\frac{d(x^2-x)^5}{dx} = 5(x^2-x)^4\frac{d(x^2-x)}{dx} = 5(x^2-x)^4\cdot(2x-1).$$
$$\frac{d\dfrac{ax+b}{cx+d}}{dx} = \frac{ad-bc}{(cx+d)^2}.$$
$$\frac{d\left(\cos(x^2)+x\cos^2 x\right)^4}{dx} = 4\left(\cos(x^2)+x\cos^2 x\right)^3\left(-\sin(x^2)2x+\cos^2 x-2x\cos x\sin x\right).$$
$$\frac{d\tan(\sqrt{x^2+1})}{dx} = \sec^2(\sqrt{x^2+1})\frac{x}{\sqrt{x^2+1}}.$$
$$\frac{d\frac{\sin(x^2+1)}{\cos(x-1)}}{dx} = \frac{2x\cos(x^2+1)\cos(x-1)+\sin(x-1)\sin(x^2+1)}{\cos^2(x-1)}.$$

(18) The slope is the derivative of the function at the point given ($x=1$). That is,
$y'(x) = 2(x^2-1)2x; \quad y'(1) = 0.$

(19) Velocity is $x'(t) = \frac{20}{9}t$ meters per second. This is 0 at time 0 seconds. It is equal 60 at time $t = 27$ seconds. At that time distance is $x(t) = 810$ meters.

(20) (1) The correct answer is A. Indeed, $\lim_{x\to\infty}\cos(\frac{1}{x}) = \cos 0 = 1$ while the other limits are either equivalent to $\lim_{x\to\infty}\cos x$ (B and C), which does not exists since cosine oscillate, or it oscillates to infinity with the oscillation getting larger and larger (D).

(2) The answer is C. A and D are differentiable (use the side limits for the derivative in A and the definition of derivative as a limit in D), and B is not even continuous.

(3) The correct answer is D. If the limit in D exists, then f has a derivative at a. And if it has a derivative at a, then it is continuous at a, and therefore the limit of $f(x)$ is $f(a)$. A is not correct because f needs to be continuous to have $f(a) = 1$, and nowhere it says f is. B is also wrong since the 11th derivative is $-\cos x$. C is also incorrect, it suffices to choose $f(x) = g(x)$ to have a limit that exists and it is equal 1.