List of topics and problems for exam 2. The following is a list of topics that are included in the second midterm, together with some hints on how to approach them. All topics need a certain level of sophistication in algebra. If you have problems with algebra, please see me or the TA. Also, although we will not ask specific questions about derivatives or limits, some of the problems will require you to know how to find limits and derivatives. So be sure you know that part very well, otherwise you will lose points because of past material (and you will be marked down for these). Be sure you understand concepts, there will be some true/false questions to assess just that.

1. Implicit differentiation and related rates: This is really from last midterm, but we did not include it then so we include it here. It covers applied problems as in page 86 and chain rule as applied to implicit differentiation (calculate derivatives of functions given by an equation, rather than by a formula). An application of implicit differentiation is finding the derivatives of the inverse trig functions, which you have to know how to find, not just the formula. The arcsine is done in 15.5.

2. Chapter 5
   (a) Tangent and normal lines: know how to calculate them, it also includes some geometric problems like those in page 89 1-9.
   (b) Intermediate value theorem: this is mostly for use in graphing so one can learn when a function changes sign, but I could use it in the true-false portion of the exam (see 2.4), plus there are other applications like example 2.3 (which I did in class).
   (c) Using the intermediate value theorem we can find where a function is positive or negative. We can use it also to find where the derivative of a function is positive or negative, or where the second derivative of a function is positive or negative.
   (d) We can then use the derivative test to find if a functions is increasing or decreasing. Be sure to know which direction some properties go (if \( f \) is increasing, it does not mean \( f' > 0 \), but if \( f' > 0 \), then it is increasing). This is good material for true/false.
   (e) The mean value theorem is a very important theorem, although we did not use it much. I want you to know what it says, I could include a true/False question to check if you know what it says.
   (f) Maxima and minima: both local and absolute ones. Can you find them? (find stationary points, use the sign of the derivative to check what you have, max or min, know how to find absolute ones- including checking the end points of the domain - , etc).
   (g) Convexity and concavity. Find out where the second derivative is positive or negative (so the function is convex or concave) and find inflection points.
   (h) Finally put everything together to give an idea of the graph of a function: 1) find where the function is negative or positive; 2) find where the function is increasing or decreasing; 3) find out where the function is concave or convex; in view of the previous information find the local max and min, and the inflection points; 4) find asymptotes and sketch the graph. No need to find extra points (other than max, min and inflection) to make the graph accurate, for accuracy we go to a computer.
   (i) Applied optimization: these are word problems that will require you to construct formulas and equations, and to use what we learned before to find the max or min of the function. Be sure to use the second derivative test to check that what you got is a max or a min. No need to memorize formulas of areas or volumes, etc (except for the very basic ones - a disk, the cylinder) I will give them to you if you need them. There are many examples in page 108.
   (j) The last part is parametrized curves. You need to learn: 1) what they are and the difference with the graph of a function; 2) the definition of instantaneous velocity for a parametrized curve (different from the one for a regular graph!); 3) the slope of the tangent line (again, different from the graph definition!) and how to find horizontal and vertical tangent lines; 4) the parametric formulas for a line and for a circle; 5) the curvature, definition and (sorry, but only for this time) the formula. I could also ask to find the max of the curvature of some curve (you need to find it and then find the maximum).
(k) L'Hopital's rule: yes, you can now use it! just be sure you use it properly.

(3) Chapter 6.
   (a) Exponential function with base \( a \) \((f(x) = a^x)\), graph and limit properties at infinity for both \( a > 1 \) and \( a < 1 \). Properties of the exponential (getting the properties wrong is a serious offense!).
   (b) Logarithms of base \( a \), graph and limit properties at infinity for \( a > 1 \) and \( a < 1 \). Properties of the logarithms (getting the properties wrong is also a serious offense!).
   (c) Derivatives of both exponentials and logarithms with any base, including using the chain rule (as in finding the derivative of \#19 or \# 6 in page 129). Definition of \( e \).
   (d) Limits involving exponentials, logarithms and powers of \( x \), as in problems 27-39 in page 129. I could also give you some graphs involving exponentials and logarithms, or max problems.
   (e) Doubling time and half life: straightforward, as in the homework.

(4) Chapter 7
   (a) Definition of Riemann sum, definite integral and what it represents. You need to understand this part, I could use true/false to check that.
   (b) First fundamental theorem of Calculus: what can I say, all fundamental theorems need to be learned. Please, know what it says and what it means. And of course, use it to find definite integrals!
   (c) Definition of indefinite integrals. Properties of integrals, learn mainly how to use them in practice. Be sure to know the main indefinite integrals in table 2, page 140 (you do not need the one of the secant).
   (d) Second fundamental theorem of Calculus: as I said, being fundamental it needs to be learned. Use it to calculate derivatives of integrals where the variable is a limit of integration, as in problems 2-8 in page 145.
   (e) Substitution for definite and indefinite integrals: use it to integrate effectively. You can expect some integrals where you will need to use this method, it is an important method.

(5) Chapter 8
   (a) Areas between graphs: can you find the area between two graphs?