List of topics and problems for the exam. The following is a list of topics that are included in the exam, together with some hints on how to approach them. All topics need a certain level of sophistication in algebra. If you have problems with algebra, please see me or the TA.

(1) Functions.
   (a) Definition of function, domain and range. Be able to identify a function from its graph and to find its domain and range.
   (b) Implicit functions, definition.
   (c) Inverse functions, definition and how to find them.

   Problems similar to the ones in the homework in page 19.

(2) Limits.
   (a) Intuitive and formal $\epsilon - \delta$ definition, including interpretation as the propagation of errors. Problems as #4-14 in page 35. Be sure to include a concluding remark when you explain: to make .... less than ϵ you will need δ to be ....
   (b) Side limits and the relation to limit (both side limits need to be equal for the limit to exists). Learn the formal $\epsilon - \delta$ definition of side limit even if we do not use it, and the practical way to find then using properties.
   (c) Limits at infinity. Learn the formal $\epsilon - A$ definitions of limit at $\pm\infty$, and the practical way to find them (factoring highest powers from rational functions). Be sure you know whether the limit is $+\infty$ or $-\infty$.
   (d) Properties of limits and how to use them to find limits. Include true/false problems on when the sum, product, quotient, etc, of functions exist when one of the limits fails to exist. Problems as in #1-15, 18-22, 23-25. Many of them use factorization and/or multiplication by conjugate expressions.
   (e) Trigonometric limits:
      (i) The trigonometric formulas and relations (other than the definitions of sine, cosine, tangent, secant, cosecant, etc) you need to know are all based on the unit circle and posted on the moodle page - exam week - under "Basic trig identities". I will only assume you know the formula $\cos^2 x + \sin^2 x = 1$ and the relations in the moodle page, and will provide other formulas if needed.
      (ii) There are two types of trig limits we have seen, one as $x \to 0$ and the other as $x \to \infty$. The ones using $x \to \infty$ are usually dealt with by using the sandwich theorem (you need to know what the theorem says and how to use it!). The ones using $x \to 0$ (or $x \to \pi/2$, or $x \to \pi$, etc) are dealt with using the two basic limits we know

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1; \quad \lim_{\theta \to 0} \frac{1 - \cos^2\theta}{\theta^2} = \frac{1}{2}.$$

      You will need to modify and transform your limits until you include these two in the calculations. It needs some level of sophistication in the handling of trig functions, so please practice to achieve that level! Most problems in page 54 #1-24 are of this type, and there are several variations, all of which are included in the exam.

(3) Continuity.
   (a) Definition of continuity, both formal and intuitive. Understand that being continuous at a means that one can plug in the number when taking the limit as $x \to a$.
   (b) Different types of discontinuities: jump, removable or essential. Learn how to find which type is which and how to remove removable ones.

   Problems are of the type in page 51, #26-28, and also #29-30 in page 54.

(4) Asymptotes: Learn the three different types of asymptotes and how to find them. Problems as in #1,2,3,6 in page 56.

(5) Differentiation.
   (a) Definition of derivative at a as a limit, together with the three different notations. Finding derivatives using the limit definition (no credit will be given to an answer to a problem that

specifically asks for the use of the limit definition if the answer does not use limits). Problems as #1-10 in page 63.

(b) Functions that are not differentiable: when side limits fail. Problems as in #11-14 in page 63. Remember that if the slope (or derivatives) in both sides of a point coincide, it does not mean it is differentiable, it also needs to be continuous at that point.

(c) A differentiable function is necessarily continuous (problems #15-16 in page 63).

(d) Rules of differentiation: the ones we all love! Learn the basic rules (addition, subtraction, product, quotient, power rule) and how to use them. Lots of examples in page 69.

(e) Higher derivatives, including finding the nth derivative of a basic function. As in the homework.

(f) Derivatives of trigonometric functions: mostly based on $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$ and differentiation rules, but also piecewise functions as in #12 in page 72. #1-13 in page 72.

(g) Chain rule: a more general rule than the power rule, learn how to use it effectively. Problems as in # 1-6, 9 in page 79 are good, especially #6, they mix trig with chain rule and other rules. You should expect problems where several different rules need to be applied.

(h) Related rate: applied problems as in the homework.