Math 234: Homework

Problem 1. Evaluate the surface integral:

$$\iint_S x^2yz\, dS$$

where S is the part of the plane $z = 1 + 2x + 3y$ that lies above the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$.

Problem 2. Evaluate the surface integral:

$$\iint_S xy\, dS$$

where S is the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

Problem 3. Evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ for the given $\vec{F}$ and oriented surface S.

(i) $\vec{F} = (xy, yz, zx)$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ and has the upward orientation.

(ii) $\vec{F} = (xze^y, -xze^y, z)$ and S is the part of the plane $x + y + z = 1$ in the first octant and has downward orientation.

(iii) $\vec{F} = (xz, x, y)$ and S is the hemisphere $x^2 + y^2 + z^2 = 25$, $y \geq 0$, oriented in the direction of the positive y axis.

Problem 4. Use Stokes' Theorem to evaluate $\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$.

(i) $\vec{F} = (2y\cos(z), e^x\sin(z), xe^y)$ where S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward.

(ii) $\vec{F} = (x^2z^2, y^2z^2, xyz)$, where S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylindar $x^2 + y^2 = 4$, oriented upward.

Problem 5. Use Stokes' theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

(i) $\vec{F} = (x + y^2, y + z^2, z + x^2)$, where C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ and C is given the counterclockwise orientation when viewed from above.

(ii) $\vec{F} = (yz, 2xz, e^{xy})$ where C is the circle $x^2 + y^2 = 16$, $z = 5$ and C is given the counterclockwise orientation when viewed from above.

Problem 6. This problem is not to be turned in. We will cover the material you need to do this problem on Tuesday, December 8th. This material will appear on the exam. In each of the below parts, for the given $\vec{F}$ and the given region E, verify Gauss' Theorem (the Divergence Theorem) by computing both $\iiint_E \vec{\nabla} \cdot \vec{F}\, dV$ and $\iint_S \vec{F} \cdot d\vec{S}$ (where S denotes the boundary of E with the outward orientation), and thereby showing they are equal.

(i) $\vec{F} = (3x, xy, 2xz)$ and E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

(ii) $\vec{F} = (x^2, xy, z)$ and E is the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

Problem 7. This problem is not to be turned in. We will cover the material you need to do this problem on Tuesday, December 8th. This material will appear on the exam. Use Gauss' Theorem to compute $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (e^x\sin(y), e^x\cos(y), yz^2)$ and S is the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$ with the outward orientation.