Fall Semester, 2002-03

Math 743: Exercises 2; Due Monday, October 21, 2002.

1. Let T be a linear transformation on the space of complex matrices of order n such that T preserves the spectrum (the eigenvalues, including multiplicities). Prove that there exists a nonsingular matrix P such that $T(A) = P^{-1}AP$ for all A, or $T(A) = P^{-1}A^T P$ for all A.

Hint: Argue that T preserves determinant and so is of classical form. Let eig (A) denote the n eigenvalues of A. Let $C = (T(I_n))^{-1}$. Using the characteristic matrix $\lambda I - A$, show that eig (A) = eig $(T(A))$ = eig $(CT(A))$ for all A. Conclude that eig (X) = eig (CX) for all X. Now there exists a unitary U and positive semi-definite H such that $CU = H$ (the polar decomposition) and argue that $C = I$.

2. Let T be a linear transformation on the space of complex matrices of order n. Prove that T preserves the singular values (including multiplicities) if and only if there are unitary matrices P and Q such that $T(A) = PAQ$ for all A, or $T(A) = PA^TQ$ for all A..

3. Let $A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$. Determine

a. The singular values, left singular (real) vectors, and right singular (real) vectors of A.

b. Draw a careful picture of the unit ball in $\Re^2$ and its image under A, together with the singular vectors.

c. What are the 1-, 2-, ∞-, and Frobenius norms of A?

d. The inverse of A from the SVD.