Math 641, Fall 1999
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Exercise Set 7, ** exercises due Friday, December 17, 1999

**1. Consider the code C^* obtained from a Reed-Solomon $[n = q-1, k, d = n-k+1]$ code over F_q by adding an overall parity check. As we know C^* is a $[n = q, k, n-k+1]$ code over F_q consisting of all vectors of the form

$$(f(1), f(\gamma), \ldots, f(\gamma^{q-2}), f(0))$$

where $f(x)$ ranges over all the polynomials over F_q of degree at most $k-1$.

(a) Write down a parity check matrix for C^*.

(b) Use the parity check matrix in (a) to determine a parity check matrix for an extension C^{**} of C^* which is an MDS code with parameters $[n = q+1, k, d = q-k+2]$.

**2. Let C be a binary code of length n with weight enumerator $A(z)$ and let C^* be the code of length $n+1$ obtained from C by adding an overall parity check. Prove that the weight enumerator of C^* is

$$\frac{1}{2}\left((1+z)A(z) + (1-z)A(-z)\right).$$

3. Problem 6.13.6 on page 111 of van Lint's book.

**4. Prove that the 2nd order Reed-Muller code RM(2,m) is Z_4 linear. (A hint is given in the solution for exercise 8.5.2. (p. 138) of van Lint's book on page 212.)

**5. Problem 8.5.3 on page 138 of van Lint's book. (A hint is given in the solutions on page 212.)

**6. Prove that the first order Reed-Muller code RM(1,$m+1$) of length 2^{m+1} is a subcode of the binary image of the dual of the code C_m in Example 8.4.1 (p. 136) of van Lint's book.