Math 240, Fall Semester 2001-02 NAME:
(Prof.) R.A. Brualdi Total Points:
Final Exam (150 points): 2:45 pm, December 20, 2001,
([points in brackets], calculations of factorial, binomial coefficients can be omitted)

1. [6 points] Construct the truth table for the compound statement:

$$p \text{ only if } q.$$

2. [6 points] Let $P(x,y)$ be an arbitrary predicate of two integer variables x and y.
Let $p = \forall x \exists y P(x,y)$ and let $q = \exists y \forall x P(x,y)$.

Is $p \rightarrow q$ True, False or Neither?

Is $q \rightarrow p$ True, False or Neither?

3. [10 points] Determine the number of different functions from a set of n elements to a set of m elements?

How many of these functions are one-to-one?

4. [10 points] Prove using **mathematical induction**:

$$\sum_{j=2}^{n} \binom{j}{2} = \binom{n+1}{3} \quad (n \geq 2).$$

If you use identities for binomial coefficients $\binom{n}{k}$ be sure to indicate so.

5. [10 points] Solve the recurrence relation:

$$a_n = a_{n-1} + 6a_{n-2} \quad (n \geq 2) \text{ where } a_0 = 2, a_1 = 1.$$

6. [8 points] Messages are sent over a channel using two different signals. One signal requires 2 microseconds for transmittal and the other requires 3 microseconds. Each signal of a message is followed immediately by the next signal. Find a recurrence relation for the number of different signals that can be sent in n microseconds, and give the initial conditions. (You are not expected to solve the recurrence relation.)

7. [16 points] Consider an ordinary deck of 52 cards of thirteen ranks (say, $1, 2, \ldots, 13$) and four suits (say, hearts, diamonds, spades, clubs). A **hand** means 13 (unordered) cards.

(a) How many different hands are there?

(b) How many hands contain no pairs (two cards of the same rank)?

(c) How many hands contain 5 hearts, 3 diamonds, 4 clubs, and 1 spade?

(d) What is the probability that a hand chosen at random contain 5 hearts, 3 diamonds, 4 clubs, and 1 spade?

8. [10 points] Determine the number of functions from $\{1, 2, \ldots, n\}$ to $\{a, b, c\}$ that are onto.

9. [10 points] Use the **Euclidean algorithm** to determine the GCD of $3n + 2$ and $2n + 1$ where n is an arbitrary positive integer.

10. [14 points] (a) Define an **equivalence relation**:

(b) Define a **partial order** relation:

(c) Determine the number of different equivalence relations on a set $\{a, b, c\}$ of three elements?

11. [20 points] Consider the poset whose Hasse diagram is shown. Determine, if they exist, all:

(a) maximal elements:

(b) minimal elements:

(c) all greatest elements:

(d) all least elements:

(e) the GLB of $\{a, b, c\}$:

(f) the LUB of $\{x, y\}$:

(g) whether or not the poset is a lattice:

(h) Consider the poset $(S, |)$ where $S = \{1, 2, \ldots, 100\}$. If they exist, determine for this poset:

(i) GLB of 42 and 63:

(ii) LUB of 28 and 26:

12. [10 points] Apply Dijkstra's algorithm to the weighted graph below to get shortest paths from vertex a to every other vertex in the graph):

13. [10 points] Apply Kruskal's algorithm to determine a minimum weight spanning tree for the weighted graph shown:

14. [10 points] An algebraic expression is given in preorder form as:

$$\times + \times 223 - +31 \times 52.$$

Write the expression in the usual in-order form with parantheses.