Group Theory **N. Boston**

Fall 2003 HOMEWORK 8

29. Let K be an extraspecial group of order 7^3 and exponent 7. Let a and b be generators and set $c = [a, b]$. Show that $a \mapsto a^2c, b \mapsto b^2, c \mapsto c^4$ determines an automorphism α of K of order 3, and that the semidirect product of K by $< \alpha >$ is a Frobenius group with nonabelian kernel.

30. Let G be a Frobenius group with kernel K. Show that if L is a normal subgroup of G, then either K contains L or L contains K.

HINT: Assume that K does not contain L and show that $|K|$ divides $|L|$.

31. Let p be a prime, ≥ 5. Show that if $p \equiv 3 \pmod 4$, then there are 4 isomorphism classes of groups of order $4p$, whereas if $p \equiv 1 \pmod 4$, then there are 5 of them.

32. Suppose that $Z(K) = 1$ and that $\mathrm{Out}(K)$ is solvable. Show that every extension of K by any perfect group H is isomorphic to $H \times K$.