

**9.** If  $G$  is a finite group, show that there is a nilpotent subgroup  $H$  such that  $H^G = G$ .

**10.** Show that if  $F$  is any field, then its additive group  $F^+$  is characteristically simple, but that  $\mathbf{Z}^+$  is not characteristically simple.

**11.** Show that if  $G$  is characteristically simple, then so is  $G \times G$ .

**12.** Suppose  $H$  and  $K$  are subnormal subgroups of  $G$ . Show that  $H \cap K$  is subnormal in  $G$ . Show by example that  $HK$  need not be a subgroup of  $G$  but that if  $H$  is normal in  $G$ , then  $HK$  is subnormal in  $G$ .