

5. Analyze the $2 \times 2 \times 2$ Rubik's cube in the same way we handled the $3 \times 3 \times 3$ cube. What are its composition factors?

6. The free group F on 2 generators has elements reduced words in x, x^{-1}, y, y^{-1} , where reduced means that $xx^{-1}, x^{-1}x$, etc. are equal to 1. Multiplication is juxtaposition followed by reduction. Using Dey's formula, compute the asymptotic order of $a_n(F)$, the number of subgroups of index n of F (Stirling's formula may be useful).

7. Let G be a permutation group of order p^k and degree n , where $n < p^2$. Show that G is an elementary abelian p -group.

HINT: What can you say when stabilizers are normal subgroups?

8. Show that there is no group G with derived subgroup G' isomorphic to $Sym(4)$.

HINT: Let $C = C_G(G')$. Considering inner automorphisms of $Sym(4)$, show that $G = C \times G'$.