Group Theory **N. Boston**

Fall 2003 HOMEWORK 1

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1. Let $H \subseteq G$. Show that for each prime p, $|\mathrm{Syl}_p(H)| \leq |\mathrm{Syl}_p(G)|$.

2. Let $|G| = pqr$ where $p < q < r$ are primes. Show that $|\mathrm{Syl}_r(G)| = 1$.

HINT: First prove the (easy) two prime version of this result as a lemma. Then show that if the result fails, a Sylow q-subgroup is normal and use your lemma.

3. Show that there is no simple noncyclic group of order $2^m p^n$, when $m = 1, 2,$ or 3, and p is an odd prime.

4. Show that the order of an abelian permutation group of degree n is not greater than $3^{n/3}$.

HINT: Break into transitive and intransitive cases and use induction on n in the latter case.

NOTE: There exist such groups of order 3^m and degree $3m$ for $m = 1, 2, ...$, and so the bound is sharp.