

**ECE/MATH 641: HOMEWORK 2, DUE OCT 13.**

Please solve the following problems.

1. Determine a parity check matrix for a linear binary code whose set of coset leaders is  $(000000); (100000); (010000); (001000); (000100); (000010); (000001); (110000)$ .

2. Prove that the code  $\mathcal{H}_{3,ext}$  is self-dual.

3. Let  $C$  and  $D$  be linear codes. (a) Show that  $(C^\perp)^\perp = C$ . (b) Let  $C + D = \{x + y : x \in C, y \in D\}$ . Show that  $(C + D)^\perp = C^\perp \cap D^\perp$ .

4. Let  $\mathcal{H}_4$  be the binary linear Hamming code of length  $n = 15$ .

(a) Let  $C$  be a shortening of the code  $\mathcal{H}_4$  in one coordinate. What is the number of codewords of weight 3 in the code  $C$ ?

(b) Let  $C_0$  be a puncturing of the code  $\mathcal{H}_4$  on one coordinate. What is the number of codewords of weight 3 in  $C_0$ ?

5. Let  $C$  be  $\mathcal{H}_4$ . Using MacWilliams' Theorem, find its weight enumerator. From this, find the weight enumerator of  $C_{ext}$  and of its dual  $(C_{ext})^\perp$  (a Reed-Müller code).