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CORRECTIONS AND ADDITIONS TO "SOME PROPERTIES OF MEASURE AND CATEGORY"

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It has been shown by A. Kamburelis (Wrocław) that Lemma 8.2 is false. The error in the proof occurs in the sentence "Working in $M[G]$ obtain $B_n \in \mathbf{B}^M\ldots$"; in fact, it may be impossible to obtain such B_n. He also points out that my remark following Lemma 4.2, that a finitely additive strictly positive measure is enough to prove Lemma 4.1 and Lemma 4.2, is false. This is because every σ-centered Boolean algebra, for example, the Boolean algebra associated with adding a dominating real, carries such a measure (see Kelly 1959).

Consequently, I do not know whether or not the model given for (7) works. However, an alternative model for (7) can be given. First add ω_3 side-by-side perfect set reals and then add ω_2 Cohen reals. The analog of Lemma 8.2 is true in this case. Let $\mathbf{P}$ be the countable product of perfect set forcing and let $\mathbf{C}$ be the partial order for adding one Cohen real.

LEMMA. *If $G \times H$ is $\mathbf{P} \oplus \mathbf{C}$-generic over M, then*

$$\forall f \in \omega^\omega \cap M[G, H] \exists g \in \omega^\omega \cap M[H] \forall^\infty n\, f(n) < g(n).$$

PROOF. For $p, q \in \mathbf{P}$ and $n < \omega$ define $p \leqslant_n q$ iff $p \leqslant q$ and the first n splitting nodes of q on its first n coordinates remain in p. The usual fusion lemma states that if $p_{n+1} \leqslant_n p_n$ for all $n < \omega$, then the fusion $\bigcap_{n<\omega} p_n$ is an element of $\mathbf{P}$.

Claim. Suppose $(p, r) \in \mathbf{P} \oplus \mathbf{C}$, $n < \omega$, and $(p, r) \Vdash$"$\tau \in \omega$", then there are $\hat{r} \leqslant r$, $\hat{p} \leqslant_n p$, and $N < \omega$ such that

$$(\hat{p}, \hat{r}) \Vdash \text{“}\tau < N\text{”}.$$

The proof is to successively extend r and p 2^{n^2} times. To prove the lemma, suppose

$$\Vdash_{\mathbf{P}\oplus C} \text{“}\tau \in \omega^\omega \text{ is strictly increasing”}.$$

Let $\{r_n : n < \omega\}$ be a list of all elements of $\mathbf{C}$. Working in M, build a sequence $(p_n, \hat{r}_n)$ in $\mathbf{P} \oplus \mathbf{C}$ and $f \in \omega^\omega$ such that $\hat{r}_n \leqslant r_n$, $p_n \leqslant_n p_{n-1}$ and $(p_n, \hat{r}_n) \Vdash$"$\tau(n) < f(n)$". Let p be the fusion of the p_n and let $X = \{n \mid \hat{r}_n \in H\}$ ($X \in M[H]$ is infinite). Then

$$(p, \phi) \Vdash \text{“}\forall n \in X\, \tau(n) < f(n)\text{”}.$$

Letting $g(n) = f(k_n)$ where k_n is the least element of X greater than n we have that $\forall n\, \tau(n) < g(n)$. □

Lemma 8.5 is also false, for the same reason, although it may be true if "finitely additive" is replaced by "countable additive". The models which use this lemma (9, 10, 15, 16, 21, 22) may be correct, however I do not have a proof that they are.

The second to last sentence on p. 110 should read, "In fact, having $\mathbf{B}^M$ be countable in N gives a counterexample." On p. 106 the eighth line should read, "is an unpublished result of Prikry."

Recently we have shown that $A(m)$ implies D. A corollary to this is that $A(m) + B(c)$ implies $A(c)$. Also we have found a characterization of $B(c)$ which is dual to that for $U(c)$. These results will appear elsewhere.

The bias expressed in the problem section is the author's and is not necessarily shared by the originator of the problem. The following problems should be adjoined.

(10) Suppose $M \subseteq N$ are models of ZFC. Then can forcing with $\mathbf{B}^M$ over N add an eventually dominating real? Same question for $\mathbf{E}^M$.

(11) Show that $\omega_2 \leqslant \kappa_B < \kappa_U$ is consistent.

(12) (M. Gavalec, communicated by A. Kamburelis) Is the Boolean algebra for adding a Cohen real followed by a random real isomorphic to the Boolean algebra for adding a random real and then a Cohen real?

References

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