and Loeb present a simplified language, which, though a bit awkward, suffices to handle the basic results on real topology and calculus, which they present (as is customary) as an introduction to non-standard analysis. The proper development of logic for non-standard analysis is given in the second chapter. The technicalities of actually constructing non-standard models are downplayed. This seems proper, since such constructions are part of model theory rather than non-standard analysis.

Hurd and Loeb's book is well written and organized, and it is provided with a good selection of exercises, mostly easy to moderately difficult. It is a good book from which to learn non-standard analysis.

On the whole, Stroyan and Luxemburg's book is a more advanced introduction, although some of the early chapters are designed for the reader new to non-standard analysis. Normally it presents the most general result or proof rather than the simplest or most common. It provides a more varied and deeper view of non-standard analysis and its applications, and gives a more complete treatment of the various types of saturation-like properties that can be useful in a non-standard model, with details of the construction of models that have them. It is a good second book about non-standard analysis for someone who understands the main points and is interested in learning more. The greater generality gives it value as a reference, and the wide variety of applications and general explication of analytic notions via non-standard analysis make for interesting reading.

The major topic of non-standard analysis that is missing from Stroyan and Luxemburg is measure theory. The theory of Loeb spaces was only starting out when this book was written. Although these measure-theoretic directions have seemed to many of us to be the most active part of non-standard analysis in recent years, recent work of Marc Diener and others on differential equations and Hirshfeld's non-standard proof of the Gleason–Montgomery–Zippin theorem show that there is plenty going on in the areas covered by Stroyan and Luxemburg. In any case, Hurd and Loeb have a very strong introduction to Loeb spaces, and to give a fuller treatment would require a book to itself. Such is Stroyan and Bayod, ***Foundations of infinitesimal stochastic analysis*** (LIII 1261).

Finally, I would like to make some suggestions for the next book on non-standard analysis. First, the subject should be developed more axiomatically, along the lines of internal set theory, or of the "external set theory" that is said to be forthcoming. Another advantage of the IST approach is the psychological one that the "non-standard" reals are considered ordinary, and the "standard" reals form a distinguished subset. Material on model-theoretic foundations could be left to an appendix, since the actual constructions are irrelevant to practice: only the saturation-like properties (saturation, comprehensiveness, and enlargement) of the models constructed matter. As far as foundations themselves go, I suggest that the ultraproduct construction be given up in favor of model-theoretic constructions using compactness, completeness, and elementary chains. This seems more flexible and less technical, and it puts the really important thing, the compactness theorem (which is proof-theoretically obvious) out front.

The fact that the meaning of internal set is that it is a member of a given model of set theory, and external set that it is not member of that model, should be made clear. Neither of the books under review do this. Making this clear should help avoid the confusion that this distinction commonly causes newcomers to non-standard analysis.

When one gives the non-standard development of a field, one usually begins by showing that various non-standard notions are equivalent to various standard ones. If non-standard analysis is useful in an area, it may be that it expresses the concepts better or makes them more obvious than the standard one. An example of this is Cauchy's theorem that the limit of a convergent sequence of continuous functions is continuous. The natural non-standard notion of convergence for functions turns out to be the same as the classical notion of uniform convergence. It would be an interesting experiment to develop a field using only non-standard analysis, without reference to the classical notions. It appears that Nelson's book, ***Radically elementary probability theory*** (Annals of mathematics studies, no. 117, Princeton University Press, 1987) will do this.

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Saharon Shelah. *Can you take Solovay's inaccessible away?* ***Israel journal of mathematics***, vol. 48 (1984), pp. 1–47.

Jean Raisonnier. *A mathematical proof of S. Shelah's theorem on the measure problem and related results.* Ibid., pp. 48–56.

Except for Cohen's initial proof of the independence of the continuum hypothesis, perhaps no work using forcing is better known than Solovay's theorem that it is consistent with ZF + DC that every set of reals is Lebesgue measurable (LM). In his paper (*A model of set-theory in which every set of reals is Lebesgue measurable*, XXXVIII 529) Solovay also showed that in the same model of set theory every set of reals has the Baire property (BP) and every uncountable set of reals contains a perfect subset (P). To prove his result he needed to assume the existence of an inaccessible cardinal. In fact, this is necessary for P since P implies that ω_1 is inaccessible in L, Gödel's constructible sets (Ernst Specker, *Zur Axiomatik der Mengenlehre*, XXIV 226). But in the case of LM and BP it was unknown whether an inaccessible was necessary.

The title of Shelah's paper is particularly apt as he shows that an inaccessible is necessary for the consistency of LM but not for BP. He shows that ZF + DC + LM implies that ω_1 is inaccessible in L. On the other hand, Shelah shows that any model of ZF has a generic extension in which BP holds. While the second result is technically brilliant, important, and likely to be useful for getting other results, the first result is pure magic. I can see no good reason (other than the proof) for why it should be true. A shorter proof of the necessity of inaccessibles to get LM is given in Raisonnier's paper. Nevertheless I think it would be well worth the time to look for still other proofs or plausibility arguments.

Shelah, himself, at first thought that an inaccessible was necessary to get the result for BP and he generalized this "proof" to the harder case of LM. After sitting down to write the paper he realized there were problems with the easy case: the Cohen reals. He fixed this by proving the *exact opposite* result! To obtain the consistency of BP, Shelah introduces a notion of "sweet" forcing that he iterates and then proves is nice enough to carry out the basic Solovay argument. The basic Solovay argument has been exploited by Fenstad and Normann (*On absolutely measurable sets*, ***Fundamenta mathematicae***, vol. 81 (1974), pp. 91–98) to show that absolutely Δ^1_2 sets are Lebesgue measurable. This result was also obtained by Solovay but unpublished. To illustrate the basic Solovay argument let us prove the classical result that every Σ^1_1 set has the property of Baire.

Suppose A is a Σ^1_1 set of real numbers. Then there exists a Σ^1_1 formula $\theta(x)$ with a real parameter that defines the set A. By using the reflection principle there exists a countable transitive model M of a large fragment of ZFC such that M contains the real parameter of $\theta(x)$. Consider forcing with open intervals with rational end points. If r_G is a name for the generic real, then by working in M we can find a sequence of open intervals $(I_n: n \in \omega)$ such that if $U = \bigcup\{I_n: n \in \omega\}$, then for every generic G over M

$$M[G] \models \theta(r_G) \text{ iff } (\exists n \in \omega)(I_n \Vdash \theta(r_G)) \text{ iff } (\exists n \in \omega)(r_G \in I_n) \text{ iff } r_G \in U.$$

If M is a model of a sufficiently large fragment of ZFC, then $M[G]$ will be absolute for Σ^1_1 formulas, so that $M[G] \models \theta(r_G)$ iff $r_G \in A$. Since M is countable, the set of reals *not* generic over M is meager, from which it follows that $A = U$ except on a meager set, and so A has the Baire property. For LM the measure algebra is used, i.e., random real forcing. In the more complex cases this basic argument cannot use absoluteness since it may no longer hold. Instead the homogeneity of the forcing is used.

Mathias showed that in Solovay's model every subset of $[\omega]^\omega$ is completely Ramsey, i.e., for any set $B \subseteq [\omega]^\omega$ there exists $Z \in [\omega]^\omega$ such that either $[Z]^\omega \subseteq B$ or $[Z]^\omega \cap B = \varnothing$ (*Happy families*, ***Annals of mathematical logic***, vol. 12 (1977), pp. 59–111). Mathias forcing is used instead of Cohen or random real forcing.

The bulk of Shelah's argument is that the complete Boolean algebra he gets using the iteration of sweet forcing is sufficiently homogeneous to allow a modification of Solovay's basic argument to go through. Sweet forcing has been exploited by Jacques Stern (*Regularity properties of definable sets of reals*, ***Annals of pure and applied logic***, vol. 29 (1985), pp. 289–324) to get some other nice results involving the Ramsey property.

Shelah's paper also contains a proof that adding one Cohen real adds a Souslin tree. Other proofs of this have been found by Stevo Todorčević (*Partitioning pairs of countable ordinals*, reviewed below) and Mark Bickford (see D. Velleman, *Souslin trees constructed from morasses*, ***Axiomatic set theory***, Contemporary mathematics, vol. 31, American Mathematical Society, 1984, pp. 219–241). An earlier result along this line had been obtained by Judith Roitman (*Adding a random or a Cohen real: topological consequences and the effect on Martin's axiom*, ***Fundamenta mathematicae***, vol. 103 (1979), pp. 47–60). She showed that MA fails after adding one Cohen real.

The existence of a Souslin tree in the Cohen extension shows that MA + not CH + "BP in L[R]" implies that ω_1 is inaccessible in L (L. Harrington and S. Shelah, *Some exact equiconsistency results in set theory*, ***Notre Dame journal of formal logic***, vol. 26 (1985), pp. 178–188). Just the opposite state of affairs obtains for measure. Richard Laver (*Random reals and Souslin trees*, ***Proceedings of the American***

Mathematical Society, vol. 100 (1987), pp. 531–534) has shown that by starting with a model of MA_{ω_1} and then forcing with a measure algebra, in the extension, every Aronszajn tree is special.

As further evidence of his enormous prolificacy, Shelah shows that the results of Magidor and Malitz (*Compact extensions of* $L(\mathbf{Q})$, L 1076) that they proved from $\diamondsuit_{\omega_1}$ hold in the model obtained by adding one Cohen real. Todorčević noted that this result refutes a claim of Roitman (op. cit.) that adding a Cohen real preserves $MA_{\sigma\text{-linked}}$. Cohen real forcing does preserve $MA_{\sigma\text{-centered}}$.

Another result indicating the non-duality of measure and category is the theorem that if Lebesgue measure is κ-additive (i.e., the union of fewer than κ sets of measure zero has measure zero), then the same is true for the ideal of meager sets (the union of fewer than κ meager sets is meager). This result was obtained independently by Tomek Bartoszyński (*Additivity of measure implies additivity of category*, ***Transactions of the American Mathematical Society***, vol. 281 (1984), pp. 209–213) and a little later by Raisonnier and Stern (*The strength of measurability hypotheses*, ***Israel journal of mathematics***, vol. 50 (1985), pp. 337–349). Raisonnier and Stern explicitly cite Shelah's paper as leading to the ideas behind their proof. They also show that if every Σ^1_2 set is Lebesgue measurable, then every Σ^1_2 set has the property of Baire. Note that although LM has greater consistency strength than BP it does not imply BP (S. Shelah, *On measure and category*, ***Israel journal of mathematics***, vol. 52 (1985), pp. 110–114).

Perhaps the strongest regularity property of this sort is the axiom of determinacy (AD). AD implies LM, BP, P, and much more. Solovay conjectured that a suitable large cardinal axiom implies that AD holds in $L[R]$ (see John Truss, *Models of set theory containing many perfect sets*, ***Annals of mathematical logic***, vol. 7 no. 2-3 (1974), pp. 197–219). John Steel and Donald A. Martin have shown, building on some earlier work of Woodin and Shelah, that if there is a supercompact cardinal, then AD holds in $L[R]$ (*Projective determinacy*, ***Proceedings of the National Academy of Sciences of the United States of America***, vol. 85 (1988), pp. 6582–6586).

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STEVO TODORČEVIĆ. *Forcing positive partition relations.* ***Transactions of the American Mathematical Society***, vol. 280 (1983), pp. 703–720.

STEVO TODORCEVIC. *Directed sets and cofinal types.* Ibid., vol. 290 (1985), pp. 711–723.

STEVO TODORČEVIĆ. *Reals and positive partition relations.* ***Logic, methodology and philosophy of science VII, Proceedings of the Seventh International Congress of Logic, Methodology and Philosophy of Science, Salzburg, 1983***, edited by Ruth Barcan Marcus, Georg J. W. Dorn, and Paul Weingartner, Studies in logic and the foundations of mathematics, vol. 114, North-Holland, Amsterdam, New York, Oxford, and Tokyo, 1986, pp. 159–169.

STEVO TODORČEVIĆ. *Remarks on chain conditions in products.* ***Compositio mathematica***, vol. 55 (1985), pp. 295–302.

STEVO TODORČEVIĆ. *Remarks on cellularity in products.* Ibid., vol. 57 (1986), pp. 357–372.

STEVO TODORCEVIC. *Partition relations for partially ordered sets.* ***Acta mathematica***, vol. 155 (1985), pp. 1–25.

STEVO TODORCEVIC. *Partitioning pairs of countable ordinals.* Ibid., vol. 159 (1987), pp. 261–294.

Almost everyone is aware of Ramsey's theorem in one form or another: if the pairs of an infinite set A are partitioned into two sets then one of the two sets contains all pairs of some infinite subset of A. Sierpiński showed that *infinite* could not be replaced by *uncountable* in the statement. Since then, what might be called the Ramsey theory for the uncountable has steadily gained in importance and influence. Besides the fact that it is very basic research in its own right, the growth in the study of partition relations (as they are known) can also be attributed to the number and variety of the applications of both positive and negative results to areas such as analysis, model theory, forcing, and especially (it seems) set-theoretic or general topology (which is the reviewer's field). The simplest and most common type of partition relation studied is expressed by the symbols $\alpha \to (\beta, \gamma)^2$, where α, β, and γ are cardinal numbers, which is said to hold (in graph-theoretic terms) if whenever the edges of the complete graph on a set of cardinality α are partitioned into two pieces, either there is a complete subgraph on β vertices in the first piece or a complete subgraph on γ vertices in the second. Therefore, Ramsey's theorem can be expressed by saying that the partition relation $\omega \to (\omega, \omega)^2$ holds and Sierpiński's result is that $2^\omega \to (\omega_1, \omega_1)^2$ fails. One might also think of $\alpha \to (\beta, \gamma)^2$ as asserting that whenever the pairs of α are partitioned into two pieces either there is a "square" of size β contained in the first piece or a square of size γ contained in the second.

There have been two approaches taken in studying Ramsey's theory for the uncountable. One approach is to increase the parameter α until it can be shown that β and γ can be uncountable (or even as