The following paper of Goldstern and Klausner answers the question from footnote 7 in the proof of Lemma 4.2 from my paper: Long Borel hierarchies Math Logic Quarterly, 54(2008), 301-316.

Two Simple Facts about Non-AC Forcing Martin Goldstern, Lukas Daniel Klausner (Submitted on 6 Jun 2017)
https://arxiv.org/pdf/1706.01708.pdf
We make two simple observations about properties of forcing notions without AC, one of which answers an open question. – question asked p.314 - does forcing with fin(X,2) make two sets with different cardinal in the ground model have the same cardinality in the generic extension