

Some references on wqo, bqo, scattered types, Fraisse conjecture.

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Abraham, Uri; Bonnet, Robert Hausdorff's theorem for posets that satisfy the finite antichain property. *Fund. Math.* 159 (1999), no. 1, 51–69. Generalizes Hausdorff's hierarchy of scattered linear order types.

Argyros, Spiros A.; Todorcevic, Stevo Ramsey methods in analysis. Advanced Courses in Mathematics. CRM Barcelona. Birkhauser Verlag, Basel, 2005. viii+257 pp. ISBN: 978-3-7643-7264-4; 3-7643-7264-8 Todorcevic surveys bqo theory.

Bonnet, Robert; Rubin, M. Elementary embedding between countable Boolean algebras. *J. Symbolic Logic* 56 (1991), no. 4, 1212–1229. Countable models of a complete theory of boolean algebras are wqo under elementary embeddability.

Cholak, Peter; Marcone, Alberto; Solomon, Reed Reverse mathematics and the equivalence of definitions for well and better quasi-orders. *J. Symbolic Logic* 69 (2004), no. 3, 683–712.

Cloet, P. The metamathematics of scattered linear orderings. *Arch. Math. Logic* 29 (1989), no. 1, 9–20.

Cloet, P. The metamathematics of Fraisse's order type conjecture. Recursion theory week (Oberwolfach, 1989), 41–56, Lecture Notes in Math., 1432, Springer, Berlin, 1990.

Corominas, E. On better quasi-ordering countable trees. Special volume on ordered sets and their applications (L'Arbresle, 1982). *Discrete Math.* 53 (1985), 35–53.

Diestel, Reinhard Relating subsets of a poset, and a partition theorem for WQOs. *Order* 18 (2001), no. 3, 275–279.

Duparc, J. The Steel hierarchy of ordinal valued Borel mappings. *J. Symbolic Logic* 68 (2003), no. 1, 187–234.

Dzamonja, Mirna; Thompson, Katherine A poset hierarchy. *Cent. Eur. J. Math.* 4 (2006), no. 2, 225–241. Generalizes Abraham-Bonnet (1999).

E.Ellentuck, A new proof that analytic sets are Ramsey, *J. Symbolic Logic*, 39(1974),163-165.

van Engelen, Fons; Miller, Arnold W.; Steel, John Rigid Borel sets and better quasi-order theory. Logic and combinatorics (Arcata, Calif., 1985), 199–222, Contemp. Math., 65, Amer. Math. Soc., Providence, RI, 1987.

Fraïssé, Roland Theory of relations. Revised edition. With an appendix by Norbert Sauer. Studies in Logic and the Foundations of Mathematics, 145. North-Holland Publishing Co., Amsterdam, 2000. ii+451 pp. ISBN: 0-444-50542-3 Revised version contains bqo theory.

Friedman, Harvey; Robertson, Neil; Seymour, Paul The metamathematics of the graph minor theorem. Logic and combinatorics (Arcata, Calif., 1985), 229–261, Contemp. Math., 65, Amer. Math. Soc., Providence, RI, 1987.

Friedman, Harvey M. Internal finite tree embeddings. Reflections on the foundations of mathematics (Stanford, CA, 1998), 60–91, Lect. Notes Log., 15, Assoc. Symbol. Logic, Urbana, IL, 2002.

F.Galvin and K.Prikry, Borel sets and Ramsey's theorem, J. Symbolic Logic, (1973)38, 193-198.

Hausdorff, F.; Grundzuge einer Theorie der geordneten Mengen. (German) Math. Ann. 65 (1908), no. 4, 435–505. Hierarchy of scattered linear orders.

Haussler, David Another generalization of Higman's well-quasi-order result on  $\Sigma^*$ . Discrete Math. 57 (1985), no. 3, 237–243.

Jancar, Petr A note on well quasi-orderings for powersets. Inform. Process. Lett. 72 (1999), no. 5-6, 155–160.

Jenkyns, T. A.; Nash-Williams, C. St. J. A. Counterexamples in the theory of well-quasi-ordered sets. 1969 Proof Techniques in Graph Theory (Proc. Second Ann Arbor Graph Theory Conf., Ann Arbor, Mich., 1968) pp. 87–91 Academic Press, New York

Krz, Igor Proving a witness lemma in better-quasiordering theory: the method of "extensions". Math. Proc. Cambridge Philos. Soc. 106 (1989), no. 2, 253–262.

Krz, Igor Well-quasiordering finite trees with gap-condition. Proof of Harvey Friedman's conjecture. Ann. of Math. (2) 130 (1989), no. 1, 215–226.

Kriz, Igor; Thomas, Robin On well-quasi-ordering finite structures with labels. Graphs Combin. 6 (1990), no. 1, 41–49.

Kriz, Igor; Thomas, Robin Ordinal types in Ramsey theory and well-partial-ordering theory. Mathematics of Ramsey theory, 57–95, Algorithms Combin., 5, Springer, Berlin, 1990.

- Kriz, Igor; Sgall, Jiri Well-quasiordering depends on the labels. *Acta Sci. Math. (Szeged)* 55 (1991), no. 1-2, 59–65.
- Kriz, Igor; Thomas, Robin Analyzing Nash-Williams' partition theorem by means of ordinal types. *Directions in infinite graph theory and combinatorics* (Cambridge, 1989). *Discrete Math.* 95 (1991), no. 1-3, 135–167.
- Kriz, Igor The structure of infinite Friedman trees. *Adv. Math.* 115 (1995), no. 1, 141–199.
- Kruskal, Joseph B. The theory of well-quasi-ordering: A frequently discovered concept. *J. Combinatorial Theory Ser. A* 13 (1972), 297–305.
- Kruskal, J. B. Well-quasi-ordering, the Tree Theorem, and Vazsonyi's conjecture. *Trans. Amer. Math. Soc.* 95 1960 210–225.
- Kuhn, Daniela On well-quasi-ordering infinite trees - Nash-Williams's theorem revisited. *Math. Proc. Cambridge Philos. Soc.* 130 (2001), no. 3, 401–408.
- Landraitis, Charles K. Definability in well quasi-ordered sets of structures. *J. Symbolic Logic* 42 (1977), no. 2, 289–291.
- Landraitis, Charles A combinatorial property of the homomorphism relation between countable order types. *J. Symbolic Logic* 44 (1979), no. 3, 403–411.
- Laver, Richard On Fraïssé's order type conjecture. *Ann. of Math.* (2) 93 1971 89–111.
- Laver, Richard An order type decomposition theorem. *Ann. of Math.* (2) 98 (1973), 96–119.
- Laver, Richard Well-quasi-orderings and sets of finite sequences. *Math. Proc. Cambridge Philos. Soc.* 79 (1976), no. 1, 1–10. First section surveys wqo theory.
- Laver, Richard Better-quasi-orderings and a class of trees. *Studies in foundations and combinatorics*, pp. 31–48, *Adv. in Math. Suppl. Stud.*, 1, Academic Press, New York-London, 1978.
- Louveau, Alain; Simpson, Stephen G. A separable image theorem for Ramsey mappings. *Bull. Acad. Polon. Sci. Sr. Sci. Math.* 30 (1982), no. 3-4, 105–108.
- Louveau, Alain(F-PARIS6-E); Saint-Raymond, Jean(F-PARIS6-E) On the quasi-ordering of Borel linear orders under embeddability. *J. Symbolic Logic* 55 (1990), no. 2, 537–560.
- Mathias, A. R. D. Happy families. *Ann. Math. Logic* 12 (1977), no. 1, 59–111.

- Marcone, Alberto Borel quasi-orderings in subsystems of second-order arithmetic. *Ann. Pure Appl. Logic* 54 (1991), no. 3, 265–291.
- Marcone, Alberto Foundations of BQO theory. *Trans. Amer. Math. Soc.* 345 (1994), no. 2, 641–660.
- Marcone, Alberto The set of better quasi orderings is  $\Pi_2^1$ . *Math. Logic Quart.* 41 (1995), no. 3, 373–383.
- Marcone, Alberto On the logical strength of Nash-Williams' theorem on transfinite sequences. *Logic: from foundations to applications* (Staffordshire, 1993), 327–351, Oxford Sci. Publ., Oxford Univ. Press, New York, 1996.
- Marcone, Alberto Fine analysis of the quasi-orderings on the power set. *Order* 18 (2001), no. 4, 339–347 (2002).
- Marcone, Alberto Wqo and bqo theory in subsystems of second order arithmetic. *Reverse mathematics 2001*, 303–330, Lect. Notes Log., 21, Assoc. Symbol. Logic, La Jolla, CA, 2005.
- Mekler, Alan H. Scattered subsets of  $Q$ . *Order* 9 (1992), no. 2, 159–162.
- Milner, E. C. Basic wqo- and bqo-theory. *Graphs and order* (Banff, Alta., 1984), 487–502, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 147, Reidel, Dordrecht, 1985.
- Montalban, Antonio Up to equimorphism, hyperarithmetic is recursive. *J. Symbolic Logic* 70 (2005), no. 2, 360–378.
- Montalban, Antonio Equivalence between Fraisse's conjecture andJulien's theorem. *Ann. Pure Appl. Logic* 139 (2006), no. 1-3, 1–42.
- Moore, J. Structural analysis of Aronszajn trees, to appear in the Proceedings of the 2005 Logic Colloquium in Athens, Greece. eprint 9-06 math.boisestate.edu/~justin/preprints.html Question: Are Aronszajn lines under embeddability wqo?
- Nash-Williams, C. St. J. A. On well-quasi-ordering trees. 1967 A seminar on Graph Theory pp. 79–82 Holt, Rinehart and Winston, New York. Short proof of Kruskal's Theorem.
- Nash-Williams, C. St. J. A. On well-quasi-ordering infinite trees. *Proc. Cambridge Philos. Soc.* 61 1965 697–720.
- Nash-Williams, C. St. J. A. On well-quasi-ordering transfinite sequences. *Proc. Cambridge Philos. Soc.* 61 1965 33–39. Sequences with finite range. Clopen sets are Ramsey.
- Nash-Williams, C. St. J. A. On better-quasi-ordering transfinite sequences. *Proc. Cambridge Philos. Soc.* 64 1968 273–290.
- Pouzet, Maurice Sur les premeilleurordres. (French) *Ann. Inst. Fourier (Grenoble)* 22 (1972), no. 2, 1–19.  $Q$  is bqo iff  $Q^{<\omega_1}$  is wqo.

- Pouzet, Maurice; Sobrani, Mohamed The order type of the collection of finite series-parallel posets. *Discrete Math.* 265 (2003), no. 1-3, 189–211.
- Prömel, Hans Jürgen(D-BONN-DM); Voigt, Bernd(D-BLF) From wqo to bqo, via Ellentuck's theorem. (English. English summary) Topological, algebraical and combinatorial structures. Frolík's memorial volume. *Discrete Math.* 108 (1992), no. 1-3, 83–106.
- Rado, R. Partial well-ordering of sets of vectors. *Mathematika* 1, (1954). 89–95.
- Rado, R., Erdős, P.; A theorem on partial well-ordering of sets of vectors. *J. London Math. Soc.* 34 1959 222–224.
- Robertson, Neil; Seymour, P. D. Graph minors. XX. Wagner's conjecture. *J. Combin. Theory Ser. B* 92 (2004), no. 2, 325–357.
- Rosenstein, Joseph G. Linear orderings. Pure and Applied Mathematics, 98. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, 1982. xvii+487 pp. ISBN: 0-12-597680-1
- H.Rosenthal, A characterization of Banach spaces containing  $l_1$ , *Proceedings of the National Academy of Sciences (USA)*, 71(1974), 2411-2413. Uses Galvin-Prikry Theorem.
- J.Silver, Every analytic set is Ramsey, *JSL* 35(1970), 60-64.
- Simpson, Steven BQO Theory and Fraïssé's conjecture in:  
Mansfield, Richard; Weitkamp, Galen Recursive aspects of descriptive set theory. With a chapter by Stephen Simpson. Oxford Logic Guides, 11. The Clarendon Press, Oxford University Press, New York, 1985. vii+144 pp. ISBN: 0-19-503602-6.
- Simpson, Steven Nonprovability of certain combinatorial properties of finite trees. Harvey Friedman's research on the foundations of mathematics, 87–117, *Stud. Logic Found. Math.*, 117, North-Holland, Amsterdam, 1985.
- Smoryński, C. The varieties of arboreal experience. *Math. Intelligencer* 4 (1982), no. 4, 182–189. Friedman's finite version of Kruskal's Theorem is not provable in Peano arithmetic.
- Thomas, Robin A counterexample to "Wagner's conjecture" for infinite graphs. *Math. Proc. Cambridge Philos. Soc.* 103 (1988), no. 1, 55–57.
- Thomas, Robin Well-quasi-ordering infinite graphs with forbidden finite planar minor. *Trans. Amer. Math. Soc.* 312 (1989), no. 1, 279–313.
- Thomassé, Stéphan On better-quasi-ordering countable series-parallel orders. *Trans. Amer. Math. Soc.* 352 (2000), no. 6, 2491–2505. Series-parallel means that the letter Z poset does not embed.

Todorcevic, S. Lipschitz maps on trees, report no. 13, 2000/2001 Institut Mittag-Leffler. (Eric has 5 MB pdf file) Shows there is an uncountable antichain of Aronszajn trees under tree embedability.

Wolk, E. S. Partially well ordered sets and partial ordinals. Fund. Math. 60 1967 175–186. Equivalents of wqo.