A. Miller
M873
Descriptive Set Theory
Fall 2000

Descriptive set theory goes back to the work of the French mathematicians Borel, Baire, and Lebesgue around the turn of the century who studied the basic properties of the Borel subsets of the real line, functions with the Baire property, and Lebesgue measure. In his most famous monograph Lebesgue made the error of thinking that the projection of a Borel set is Borel. This error was caught by the Russian mathematician Suslin resulting in the definition of analytic sets and leading to work by Luzin and Sierpinski, e.g., every analytic set is either countable of size continuum.

Some possible topics in Descriptive Set Theory

(Gale-Stewart) Open subsets of ω^ω are determaned.

(Wolfe) G_δ subsets of ω^ω are determaned.

(Davis) Axiom of Determanacy AD$\rightarrow$P$\rightarrow$ ω_1 is inacc in L

Banach-Mazur games, AD$\rightarrow$BP Baire Property

Becker on Banach games, Blass on long games, Prikry on Ramsey

(Mycielski-Swierczkowski) AD$\rightarrow$LM Lebesgue Measure

(Solovay) AD$\rightarrow$ ω_1 is a measurable cardinal (club filter)

Wadge's Lemma and well-foundedness of Wadge ordering

(Van Wesep-Steel) Separation

(Kechris) AD$\rightarrow$SP superperfect trees

(Kunen-Miller) compactly-Borel

Classical Desc Set Thy: Prewellordering, boundedness, union of ω_1 Borel sets, absoluteness, MA+notCH+$\omega_1 = \omega_1^L$, regularity of analytic sets

(Martin) Borel Det

(Friedman) Borel Det not provable in Z

(Martin) Analytic Det and measurable cardinals

Equivalents of $0^\sharp$

Largest thin coanalytic set, coanalytic sets under V=L

(Miller-Steel-Van Engelen) Borel sets not rigid

Solovay model for P+LM+BP, (Mathias) Ramsey, Galvin-Prikry Thm

Invariant Descriptive Set theory : Vaught transform and Lopez-Escobar Thm

(Morley) Weak Vaught's Conjecture for PC($L_{\omega_1,\omega}$) sentences

References

Classical descriptive set theory, Alexander S. Kechris, Springer-Verlag, 1995.

Recursive aspects of descriptive set theory, Richard Mansfield, Galen Weitkamp ; with a chapter by Stephen Simpson, Oxford University Press, Clarendon Press, 1985.

Descriptive set theory and forcing : how to prove theorems about Borel sets the hard way, Arnold W. Miller, Lecture notes in logic 4. Springer-Verlag, 1995.

Descriptive set theory, Yiannis N. Moschovakis, Elsevier-North Holland, 1980.

Measure and category, J.Oxtoby, Springer-Verlag, 1971.

Analytic sets, edited by C. A. Rogers ... [et al.], Academic Press, 1980.

Cabal Seminar 76-77,77-79,79-81,81-85, Lecture Notes in Mathematics, 689, 839, 1019, 1333, Springer-Verlag.