

OK

④ Prove that if

$\underline{b} > \omega$, and $\{a_\alpha \in [\omega]^\omega : \alpha < \omega_1\}$ are almost disjoint then there exists $a \in [\omega]^\omega$ with $a \cap a_\alpha$ finite for every $\alpha < \omega_1$.

define a family, \mathcal{F} , of fns ~~so that~~ so that
for $\omega \leq \alpha < \omega_1$,

$$f_\alpha(n) \in a_\alpha \cap a_n.$$

Since $a_\alpha \cap a_n$ is finite, we define $f_\alpha(n)$ to be the greatest in $a_\alpha \cap a_n$.

since this family is size ω , & $\underline{b} > \omega$, get a bounding fn. g so that

$$g >^* f_\alpha \text{ for all } \omega \leq \alpha < \omega_1.$$

~~now~~

next define a set $X = \{x_0, x_1, \dots\}$

so that $x_n \in a_n$

and x_n is the least in a_n so that

$$x_n > g(n).$$

note that we may have assumed a_0, a_1, \dots (the first ω sets in the original family) were completely disjoint by throwing away

$a_n \setminus \bigcup_{i < n} a_i \cap a_n$ \leftarrow
(a_n would still be infinite since \leftarrow is a finite union of finite stuff.)

So $|X| = \omega$.



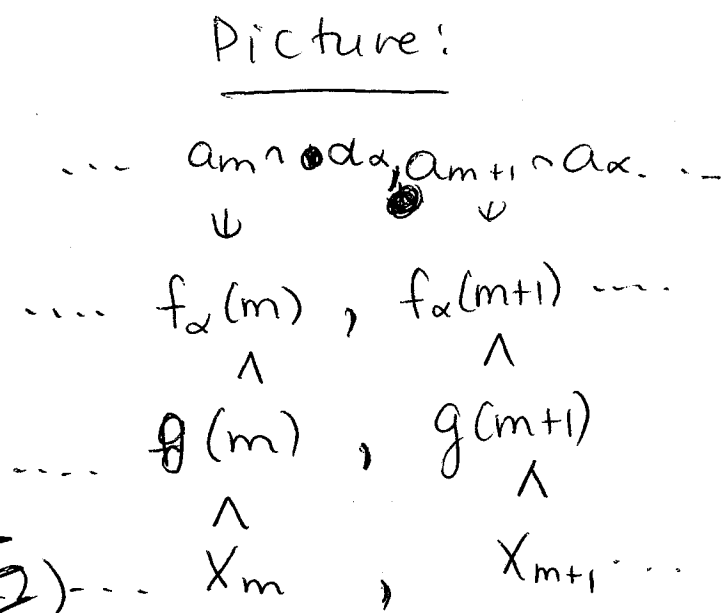
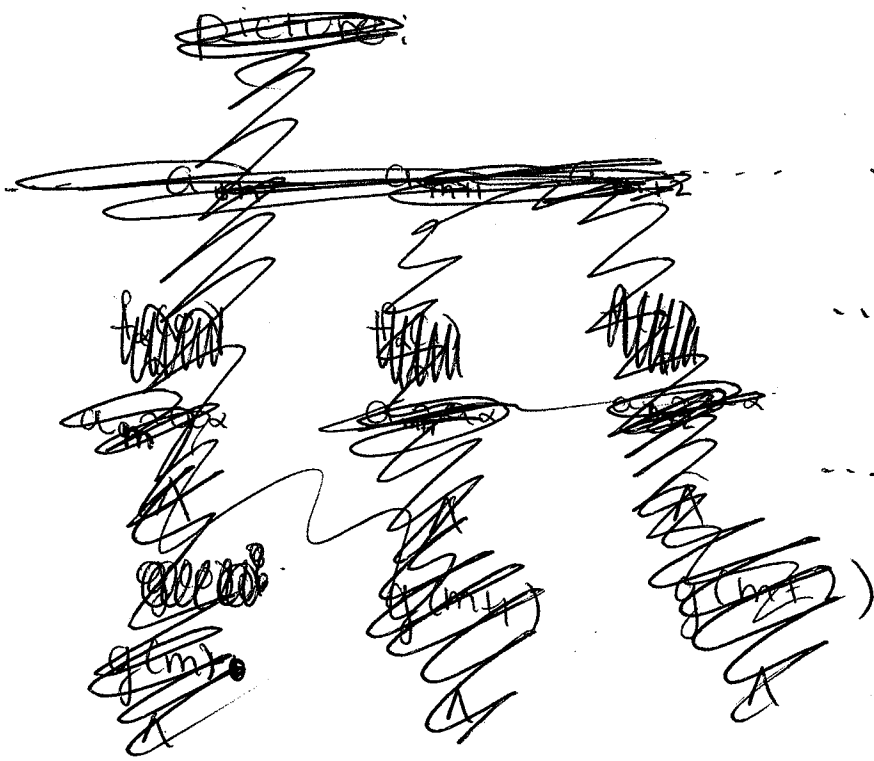
also note that

$$|X| \cap a_n = X_n \text{ for all } n < \omega$$

and

$|X| \cap a_\alpha$ is finite since there is an m , after which ~~all~~

$$X_m > g(m) > a_m \cap a_\alpha \text{ for all subsequent steps.}$$



So X is almost disjoint from the original family \square .