

1. (Zermelo) The axiom of choice AC is equivalent to every set can be well-ordered WO.
2. (Shroeder, Bernstein) If there are one-to-one maps taking X to Y and Y to X , then there is a bijection taking X to Y .
3. $V_{\omega+\omega} \models Z$.
4. ω is an ordinal.
5. Every well-ordering is isomorphic to a unique ordinal.
6. For any set X its transitive closure exists.
7. There exists a Bernstein set.
8. (Given $G : V \rightarrow V$ any class function)
There exists a unique $F : V \rightarrow V$ such that $F(\alpha) = G(F \upharpoonright \alpha)$ for every ordinal α .
9. Axiom of Foundations is equivalent to every set x is in some V_α .
10. $\text{cof}(\kappa^+) = \kappa^+$.
11. (König) $\kappa^{\text{cof}(\kappa)} > \kappa$.
12. The Downward going Lowenheim-Skolem-Tarski theorem.
13. (Vaught) Reflection Schema.
14. (X, E) is a well-founded relation iff there exists a rank function

$$r : X \rightarrow ORD$$

15. (Mostowski Collapse) If (X, E) is a well-founded extensional relation, then there exists a transitive set M and an isomorphism $j : (X, E) \rightarrow (M, \in)$.
16. Δ_0 formulas are absolute for transitive models.
17. Δ_1 formulas are absolute for transitive models of ZFC.
18. Lemma. $N = \text{Def}(M)$ is Δ_1 - ZF.
19. (Gödel 1938) $L \models \text{ZF} + \text{AC} + \text{GCH}$. $\text{Con}(\text{ZF}) \rightarrow \text{Con}(\text{ZFC} + \text{GCH})$
20. (For any large enough finite fragment of ZFC, ZFC_n)
Lemma. If M is a standard transitive model of $\text{ZFC}_n + V=L$, then $M = L_\delta$ for some ordinal δ .
21. ZF has a well-founded model iff it has a countable transitive standard model. If ZF has a model, it has one in which there is no transitive standard model.

22. (Kurepa) there exists a Souslin line iff there exists a Souslin tree.
23. Lemma. If I_α for $\alpha < \omega_1$ are intervals constructed in a Souslin line so that I_α avoids all the endpoints of I_β for $\beta < \alpha$, then reverse inclusion on these intervals makes them a Souslin tree.
24. Lemma. If there exists a Souslin tree, then there exists a nice Souslin subtree of $\mathbb{Q}^{<\omega_1}$.
25. Lemma. The lexicographical order on the maximal chains of a nice tree is a Souslin line.
26. (Tarski) Elementary Chains of substructures.
27. (Fodor) Push down lemma.
28. (Jenson) $\diamond_{\omega_1}$ implies there exists a nice Souslin tree.
29. (Jenson) $V=L$ implies $\diamond_{\omega_1}$.
30. For every $p \in \mathbb{P}$ there exists G which is $\mathbb{P}$ -generic over M and $b \in \mathbb{P}$.
31. $M[G]$ is transitive, $M \subseteq M[G]$, $G \in M[G]$, $\text{rank}(M[G]) = \text{rank}(M)$.
32. Given the Truth and Definability Lemma, $M[G]$ is a model of ZFC.
33. ccc forcing preserves cofinalities and cardinals.
34. Δ -systems lemma.
35. $\text{FIN}(\kappa, 2)$ has ccc. Forcing with it gives model of notCH.
36. G $\mathbb{P}$ -generic over M iff it is a $\mathbb{P}$ -filter which meets every maximal antichain in M .
37. Generic filters are maximal compatible sets.
38. How to remove the well-foundedness assumption from forcing proofs, e.g., reflection, forcing over nonstandard models, boolean value models, syntactical methods.
39. (MA_{ω_1}) No Souslin trees, notCH, for any $\{f_\alpha : \alpha < \omega_1\} \subseteq \omega^\omega$ there exists $f \in \omega^\omega$ such that $f_\alpha \leq^* f$ for all α .
40. (MA_{ω_1}) implies $2^\omega = 2^{\omega_1}$. (almost disjoint families)
41. (MA_{ω_1}) additivity of measure, category. ccc is productive, property K. Aronszahn trees are special.
42. $\text{Con}(\text{ZFC}) \rightarrow \text{Con}(\text{ZFC} + \text{MA}_{\omega_1})$.