A.Miller
M571
Spring 2002

Review questions for Final

1. Define theory, complete theory, axioms of a theory, and decidable theory. Prove that a complete theory with a computable set of axioms is decidable. Give an example of such a theory.

2. Let $\mathcal{N} = (\omega, +, \cdot, \leq, S, 0)$. Give a sketch of the proof that Th($\mathcal{N}$) is undecidable. What is the relationship between primitive recursive functions, recursive functions, computable functions, computably enumerable sets, and arithmetic sets? (Define all of these.)

3. Deduce as a corollary that for any computable set $\Sigma \subseteq$Th($\mathcal{N}$) that there is a sentence of arithmetic θ which is true in $\mathcal{N}$ but Σ does not prove θ.

4. What is an interpretation of a theory T into another theory T'? Prove in this case that if T is undecidable, then T' is undecidable.

Prove:

(a) Th($\mathcal{N}$) is interpretable into Th$(\omega, +, \cdot)$

(b) Th$(\omega, +, \cdot)$ is undecidable.

(c) Th$(\omega, +, \cdot)$ is interpretable in the Th$(\omega, \cdot, S)$.

(d) Th$(\omega, \cdot, S)$ is undecidable.

5. What does the Łos-Vaught test say? How is it proved? Give an example using it.

6. What is the completeness theorem of first-order logic? Give a sketch of its proof.

7. What is the Lowenheim-Skolem theorem? How is it proved?

8. What is a well-formed formula of propositional logic? Give the formal definition.

9. What is the compactness theorem for propositional logic? Give a sketch of its proof. Give an example using it.

10. Is the compactness theorem true for first-order logic? Does it have anything to do with the completeness theorem?

Final exam - B105 Van Vleck (usual classroom) - Monday May 13 2:45-4:45pm.