A.Miller
M571
Spring 2002
Final Exam

Answer any four of the following questions plus one more. The fifth problem can be another one taken from below or if you prefer you can do a homework exercise of your choice (state the exercise fully and pick one that is as hard as you are able to do).

Answer each of the five questions as fully as you can and as if you were telling it to another student not necessarily in this course.

Please use a separate sheet or sheets of paper for each problem.

1. Define theory, complete theory, axioms of a theory, and decidable theory. Prove that a complete theory with a computable set of axioms is decidable. Give an example of such a theory.

2. Let $\mathcal{N} = (\omega, +, \cdot, \leq, S, 0)$. Give a sketch of the proof that Th($\mathcal{N}$) is undecidable. What is the relationship between primitive recursive functions, recursive functions, computable functions, computably enumerable sets, and arithmetic sets? (Define all of these.)

3. Deduce as a corollary that for any computable set $\Sigma \subseteq$Th($\mathcal{N}$) that there is a sentence of arithmetic θ which is true in $\mathcal{N}$ but Σ does not prove θ.

4. What is an interpretation of a theory T into another theory T'? Prove in this case that if T is undecidable, then T' is undecidable.

Prove:
(a) Th($\mathcal{N}$) is interpretable into Th$(\omega, +, \cdot)$
(b) Th$(\omega, +, \cdot)$ is undecidable.
(c) Th$(\omega, +, \cdot)$ is interpretable in the Th$(\omega, \cdot, S)$.
(d) Th$(\omega, \cdot, S)$ is undecidable.

5. What does the Łos-Vaught test say? How is it proved? Give an example using it.

6. What is the completeness theorem of first-order logic? Give a sketch of its proof.

7. What is the Lowenheim-Skolem theorem? How is it proved?

8. What is a well-formed formula of propositional logic? Give the formal definition.

9. What is the compactness theorem for propositional logic? Give a sketch of its proof. Give an example using it.

10. Is the compactness theorem true for first-order logic? Does it have anything to do with the completeness theorem?