

Define the following concepts: (Answers have been added after “:.”)

1. Countable: A set X is countable iff it is empty or there is a map from ω onto X .
2. Uncountable: Uncountable means not countable.
3. ω_1 (or as Munkres calls it S_Ω):
This is the smallest uncountable well-ordering. It is characterized by the properties
 - (a) it is a well-ordering
 - (b) it is uncountable
 - (c) each initial segment is countable, i.e. $\{\alpha : \alpha < \beta\}$ is countable for each $\beta < \omega_1$.
4. Dense: A set $D \subseteq X$ is dense iff its closure is X .
5. Separable: A space is separable iff it contains a countable dense subset.
6. Neighborhood basis: A neighborhood basis for $x \in X$ is a family $\mathcal{B}$ of open neighborhoods of x such that for any open neighborhood U of x there exists $B \in \mathcal{B}$ with $x \in B \subseteq U$.
7. First countable: A space is first countable iff each of its points has a countable neighborhood base.
8. Second countable: A space is second countable iff it has a countable basis.
9. Lindelöf: A space is Lindelöf iff every open cover contains a countable subcover.
10. Hausdorff: A space is Hausdorff iff any two distinct points can be separated by disjoint open sets.
11. Regular: A space X is regular iff it is Hausdorff and for every closed set C and point $x \in X - C$ there exists disjoint open U, V with $x \in U$ and $C \subseteq V$. Equivalently, for every $x \in U$ open there exists open V with $x \in V \subseteq \overline{V} \subseteq U$.
12. Normal: A space is normal iff it is Hausdorff and any two disjoint closed sets can be separated by disjoint open sets.
13. n-Manifold: An n-manifold is a second countable Hausdorff space which has the property that every point has a neighborhood homeomorphic to $\mathbb{R}^n$.
14. Quotient map and Quotient space: Y is a quotient space of X iff there exists an onto map $p : X \rightarrow Y$ which satisfies for all $U \subseteq Y$:
 U is open in Y iff $p^{-1}(U)$ is open in X .
Such a p is called a quotient map.
15. (Extra Credit) Embedding or imbedding or emersion or immersion or submersion:
 X can be embedded into Y iff X is homeomorphic to a subspace of Y .