

1. Define $\Pi_{i \in I} X_i = \{ \langle x_i : i \in I \rangle : \forall i \in I \ x_i \in X_i \}$

The I -sequence $\langle x_i : i \in I \rangle$ which is also written $(x_i)_{i \in I}$, is a function with domain I which maps each $i \in I$ to an element $x_i \in X_i$.

2. If X_i for $i \in I$ are topological spaces, then what is a basic open subset of $\Pi_{i \in I} X_i$ (in the Tychonoff product topology)?

A set of the form $\Pi_{i \in I} U_i$ where $U_i \subseteq X_i$ is open for every $i \in I$ and $U_i = X_i$ for all but finitely many $i \in I$.

3. If X_i for $i \in I$ are topological spaces, then what is a basic open subset of $\Pi_{i \in I} X_i$ (in the Box product topology)?

A set of the form $\Pi_{i \in I} U_i$ where $U_i \subseteq X_i$ is open for every $i \in I$.

4. A metric d on X is a function $d : X \times X \rightarrow [0, \infty)$ such that

- (a) for all $x, y \in X$ $d(x, y) = 0$ iff $x = y$,
- (b) for all $x, y \in X$ $d(x, y) = d(y, x)$, and
- (c) for all $x, y, z \in X$ $d(x, z) \leq d(x, y) + d(y, z)$

5. If d is a metric on X and $\epsilon > 0$, then $B_\epsilon^d(x) = \{y \in X : d(x, y) < \epsilon\}$

6. If (X, d) is a metric space then $U \subseteq X$ is open
iff $\forall x \in U \ \exists \epsilon > 0 \ B_\epsilon^d(x) \subseteq U$

7. For a sequence $\langle x_n : n \in \omega \rangle$ in a space X then $\lim_{n \rightarrow \infty} x_n = x_0$ iff for all U an open neighborhood of x_0 for all but finitely many $x_n \in U$.

8. For X a topological space, $A \subseteq X$ is clopen iff A is closed and open.

9. For X a topological space, X is connected iff it contains no nontrivial clopen set. (X and the empty set are the trivial clopen sets.)

10. $\mathcal{U}$ is an open cover of X iff the elements of $\mathcal{U}$ are open and $\bigcup \mathcal{U} = X$.

11. $\mathcal{V}$ is a subcover of the cover $\mathcal{U}$ iff $\mathcal{V} \subseteq \mathcal{U}$ and $\bigcup \mathcal{V} = X$.

12. A topological space X is compact iff every open cover of X has a finite subcover.