

Complete the following definitions:

1. The cofinite topology on X has a basis which is

2. The discrete topology on X has a basis which is

3. The order topology on the linear order $(L, \leq)$ has a basis which is

4. The product topology on $X \times Y$ has a basis which is

5. The subspace topology on $Y \subseteq X$ is

6. The closure of the set $A \subseteq X$ is

7. The set of limit points of the set $A \subseteq X$ is

8. The interior of the set $A \subseteq X$ is

$$A^\circ$$

9. The set of boundary points of the set $A \subseteq X$ is

10. A topological space X is Hausdorff iff

11. If $f : X \rightarrow Y$ is a function and $B \subseteq Y$, then define $f^{-1}(B) =$

12. If $f : X \rightarrow Y$ is a function and $A \subseteq X$, then define $f(A) =$

13. If $f : X \rightarrow Y$ is a function, then f is one-to-one iff

14. If $f : X \rightarrow Y$ is a function, then f is onto iff

15. A function $f : X \rightarrow Y$ where X and Y are topological spaces is continuous iff

16. A function $f : X \rightarrow Y$ where X and Y are topological spaces is a homeomorphism iff

Answers

1. The cofinite topology on X has a basis

$$\mathcal{B} = \{A \subseteq X : X \setminus A \text{ is finite}\}$$

A natural subbasis is the set of compliments of singletons.

2. The discrete topology on X has a basis

$$\mathcal{B} = \{\{x\} : x \in X\}$$

3. The order topology on the linear order $(L, \leq)$ has a basis

$$\mathcal{B} = \{\{x \in L : a < x < b\} : a, b \in L \cup \{-\infty, \infty\}\}$$

4. The product topology on $X \times Y$ has a basis

$$\mathcal{B} = \mathcal{B}_X \times \mathcal{B}_Y$$

where $\mathcal{B}_X, \mathcal{B}_Y$ are bases for X, Y respectively.

5. The subspace topology on $Y \subseteq X$ is

$$\tau_Y = \{Y \cap U : U \in \tau_X\}$$

where τ_X is the topology on X .

6. The closure of the set $A \subseteq X$ is

$$\overline{A} = \bigcap \{C^{\text{closed}} \subseteq X : A \subseteq C\}$$

7. The set of limit points of the set $A \subseteq X$ is A' where $x \in A'$ iff for all U an open neighborhood of x there exists $y \in U \cap A$ with $y \neq x$.

8. The interior of the set $A \subseteq X$ is

$$A^\circ = \bigcup \{U^{\text{open}} \subseteq X : U \subseteq A\}$$

9. The set of boundary points of the set $A \subseteq X$ is

$$\partial A = \overline{A} \cap \overline{X \setminus A}$$

10. A topological space X is Hausdorff iff any two distinct points in X have disjoint open neighborhoods.

11. If $f : X \rightarrow Y$ is a function and $B \subseteq Y$, then define $f^{-1}(B) = \{x \in X : f(x) \in B\}$

12. If $f : X \rightarrow Y$ is a function and $A \subseteq X$, then define $f(A) = \{f(a) : a \in A\}$ or equivalently $f(A) = \{b \in Y : \exists a \in A f(a) = b\}$

13. If $f : X \rightarrow Y$ is a function, then f is one-to-one iff for all $x, y \in X$ if $f(x) = f(y)$, then $x = y$.

14. If $f : X \rightarrow Y$ is a function, then f is onto iff for every $y \in Y$ there exists $x \in X$ such that $f(x) = y$.

15. A function $f : X \rightarrow Y$ where X and Y are topological spaces is continuous iff $f^{-1}(V)$ is open in X for all V open in Y .

16. A function $f : X \rightarrow Y$ where X and Y are topological spaces is a homeomorphism iff f is one-to-one, onto, continuous, and open. A function f is open iff $f(U)$ is open in Y for all U open in X . Equivalently f is homeomorphism iff f is a bijection and both f and f^{-1} are continuous.