

Complete the following definitions:

1. If $\mathcal{F}$ is a family of sets, then $\bigcup \mathcal{F} =$

2. τ is a topology on X iff

3. The discrete topology on X is

4. The cofinite topology on X is

5. $\mathcal{B}$ is a basis for a topology on X iff

6. If $\mathcal{B}$ is a basis for a topology on X then a set $U \subseteq X$ is open iff

7. $\mathcal{S}$ is a subbase for a topology on X iff

8. $(L, \leq)$ is a linear order iff

9. The order topology on the linear order $(L, \leq)$ is

10. $X \subseteq L$ and $b \in L$ is an upper bound of X iff

11. The linear order $(L, \leq)$ has the least upper bound property iff

12. $(L, \leq)$ is a well-ordering iff

13. The product topology on $X \times Y$ is

14. The subspace topology on $Y \subseteq X$ is

15. A set $C \subseteq X$ is closed iff

16. The closure of the set $A \subseteq X$ is

Answers

1. If $\mathcal{F}$ is a family of sets, then $\bigcup \mathcal{F} = \{x : \exists F \in \mathcal{F} \ x \in F\}$.
2. τ is a topology on X iff the empty set and X are in τ , τ is closed under arbitrary unions, and τ is closed under finite intersections. Equivalently,

$$\begin{aligned} &\emptyset, X \in \tau \\ &\forall \mathcal{U} \subseteq \tau \quad \bigcup \mathcal{U} \in \tau \\ &\forall U, V \in \tau \quad U \cap V \in \tau \end{aligned}$$
3. The discrete topology on X is the topology on X in which all subsets of X are open.
4. The cofinite topology on X is the topology τ on X

$$\tau = \{U \subseteq X : U = \emptyset \text{ or } X \setminus U \text{ is finite}\}.$$
5. $\mathcal{B}$ is a basis for a topology on X iff $\bigcup \mathcal{B} = X$ and

$$\forall B_1, B_2 \in \mathcal{B} \quad \forall p \in B_1 \cap B_2 \quad \exists B_3 \in \mathcal{B} \quad p \in B_3 \subseteq B_1 \cap B_2.$$
6. If $\mathcal{B}$ is a basis for a topology on X then a set $U \subseteq X$ is open iff

$$\forall p \in U \quad \exists B \in \mathcal{B} \quad p \in B \subseteq U.$$
7. $\mathcal{S}$ is a subbase for a topology on X iff $\bigcup \mathcal{S} = X$.
8. $(L, \leq)$ is a linear order iff the following four axioms hold:

$$\begin{aligned} &\forall x \in L \quad x \leq x \\ &\forall x, y \in L \quad (x \leq y \text{ and } y \leq x) \text{ implies } x = y \\ &\forall x, y, z \in L \quad (x \leq y \text{ and } y \leq z) \text{ implies } x \leq z \\ &\forall x, y \in L \quad x \leq y \text{ or } y \leq x \end{aligned}$$
9. The order topology on the linear order $(L, \leq)$ is the topology with the basis consisting of open intervals in L (including open rays $(-\infty, a)$ and (a, ∞)).
10. $X \subseteq L$ and $b \in L$ is an upper bound of X iff $\forall x \in X \quad x \leq b$.
11. The linear order $(L, \leq)$ has the least upper bound property iff every nonempty $X \subseteq L$ which has an upper bound has an upperbound which is $\leq$ all other upper bounds.
12. $(L, \leq)$ is a well-ordering iff it is a linear order in which all nonempty subsets have a least element, i.e., $\forall X \subseteq L$ which is nonempty $\exists x \in X$ such that $\forall y \in X \quad x \leq y$.
13. The product topology on $X \times Y$ is the topology with basis

$$\mathcal{B} = \{U \times V : U^{\text{open}} \subseteq X, \ V^{\text{open}} \subseteq Y\}$$

14. The subspace topology on $Y \subseteq X$ is

$$\tau_Y = \{U \cap Y : U \in \tau_X\}$$

15. A set $C \subseteq X$ is closed iff $X \setminus C$ is open.
16. The closure of the set $A \subseteq X$ is

$$\bigcap \{C^{\text{closed}} \subseteq X : A \subseteq C\}$$