Last revised: May 2, 2014

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Practice Final

Final exam:
Mon May 12 745am-945am B115 Van Vleck

**1** *(Gauss's Lemma) Suppose* $f \in \mathbb{Z}[x]$*, then*
$f$ *is irreducible in* $\mathbb{Q}[x]$ *iff* $f$ *is irreducible in* $\mathbb{Z}[x]$*.*

**2** *(char 0) Suppose* $F \subseteq E$ *are fields and* $[E : F]$ *is finite. Then there exists* $\gamma$ *such that* $E = F[\gamma]$*.*

**3** *Suppose that* $K$ *is the splitting field of a polynomial in* $F[x]$ *of degree* $n$*. Then* $\mathrm{aut}(K|F)$ *is isomorphic to a subgroup of* $S_n$*.*

**4** *Suppose* $F \subseteq K$*,* $K$ *is a splitting field over* $F$*,* $p \in F[x]$ *is irreducible, and there is* $\alpha \in K$ *such that* $p(\alpha) = 0$*. Then* $p$ *splits in* $K$*.*

**5** *(char 0) Suppose* $F \subseteq K$*,* $K$ *is a splitting field over* $F$*. Then*

$$|\mathrm{aut}(K, F)| = [K : F]$$

**6** *Subgroups of solvable groups are solvable.*

**7** *(char 0) Suppose* $F \subseteq K$ *and* $K$ *is the splitting field of a polynomial in* $F[x]$ *and* $H \subseteq \mathrm{aut}(K|F)$ *is a subgroup. Then there exists a field* $E$ *with* $F \subseteq E \subseteq K$ *and* $\mathrm{aut}(K|E) = H$*.*

**8** *Every square matrix with entries in an algebraically closed field is similar to an upper triangular matrix.*