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Math 542
ok

33. Does $\{(1234), (13)\}$ generate S_4? must n be prime in the lemma?

pf: Let $\sigma = (1234)$, $\tau = (13)$

	$\xrightarrow{\sigma}$	$\xrightarrow{\tau}$	$\xrightarrow{\sigma}$	$\xrightarrow{\tau}$
1	4	2	3	1
2	1	1	2	2
3	2	4	1	3
4	3	3	4	4

$\Rightarrow \sigma\tau\sigma\tau = id$

$\Rightarrow \sigma\tau = \tau^{-1}\sigma^{-1}$

We have that $\sigma^4 = id$, $\tau^2 = id$

$\Rightarrow \sigma^{-1} = \sigma^3$, $\tau^{-1} = \tau$

$\Rightarrow \sigma\tau = \tau\sigma^3$

So the set generated by $\{(1234), (13)\}$ has presentation
$\langle \sigma, \tau \mid \sigma^4 = \tau^2 = 1, \sigma\tau = \tau\sigma^3 \rangle$

This is precisely the presentation for D_4

Since $|D_4| < |S_4|$, it doesn't generate S_4

$\Rightarrow$ n has to be prime

$\square$

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MATH 542 HOMEWORK DUE April 23

Problem 33 In the lemma 10.14 (d) must n be prime?
Prove or disprove $\{(1,2,3,4), (1,3)\}$ generates S_4.

$\{(1,2,3,4), (1,3)\}$ doesn't generate S_4.

Since $\langle (1,2,3,4), (1,3) \rangle = \{(1), (1,3), (2,4), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3), (1,2,3,4), (1,4,3,2)\} \neq S_4$, we know that $\{(1,2,3,4), (1,3)\}$ doesn't generate S_4.

In conclusion, in the Lemma 10.14(d), n must be prime.