Math 542 4/7 HW

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30 Suppose that $F \subseteq K_1 \subseteq L$ and $F \subseteq K_2 \subseteq L$ and K_1 and K_2 are splitting fields over F. Prove that $K_1 \cap K_2$ is a splitting field over F

Any irreducible polynomial $p(x) \in F[x]$ which has a root in $K = K_1 \cap K_2$ splits:

Pf: Let $p(\alpha) = 0$ with $\alpha \in K$. Then $\alpha \in K_1$ and $\alpha \in K_2$, so since K_1 and K_2 are splitting fields, p splits in both K_1 and K_2.

Therefore all of the roots of p are in K_1 and K_2 so they are in K and p splits in K.

K is a finite extension since both K_1 and K_2 are, so $K = F[\alpha_1, \dots, \alpha_n]$. Let f_i be the minimal polynomial of α_i over F. Then by the above, f_i splits in K.

Now K is the splitting field of $f_1(x) f_2(x) \cdots f_n(x) \in F[x]$.