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Homework 22

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Problem 1 : *(Wed Apr 2) Suppose* $[F[\alpha]:F]=n$, $[F[\beta]:F]=m$, *and* $gcd(n,m)=1$. *Prove that* $[F[\alpha,\beta]:F]=nm$.

Problem 1 Solution: First we will prove that $[F[\alpha,\beta]:F]\leq nm$. By *theorem 42* we can get:

$$[F[\alpha,\beta]:F]=[F[\alpha][\beta]:F[\alpha]][F[\alpha]:F]=n[F[\alpha][\beta]:F[\alpha]] \tag{1}$$

Then suppose $f(x)\in F[x]$ is irreducible and $f(\beta)=0$. Then by definition $deg(f)=m$. But it is obvious that $f(x)\in F[\alpha][x]$. So we can conclude that $[F[\alpha][\beta]:F[\alpha]]\leq m$. So in conclusion :

$$[F[\alpha,\beta]:F]\leq mn$$

But on the other hand, by the equation 1 we know that $n|[F[\alpha,\beta]:F]$. Symmetrically: $m|[F[\alpha,\beta]:F]$. So:

$$mn|[F[\alpha][\beta]:F[\alpha]]$$

In conclusion:

$$[F[\alpha,\beta]:F]=mn$$