Math 542 Exercises 21

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Power rule

I'll use proof by induction. For the base case, let $n = 2$. We have, by theorem 36 (product rule),

$$(f^2)' = (f \cdot f)' = ff' + f'f = 2ff' = 2f^{2-1}f^1$$

Now, assume for induction that $(f^n)' = n(f^{n-1})f'$. Then, using this hypothesis and the product rule:

$$(f^{n+1})' = (f^n f)' = nf^{n-1}f'f + f^n f' = nf^n f' + f^n f' = (n+1)f^n f'$$

Chain rule

Let $f = a_0 + a_1 x + \cdots + a_n x^n$. Then $f \circ g = a_0 + a_1 g + a_2 g^2 + \cdots + a_n g^n$. Then using the power rule from the previous part, we can get

$$(f \circ g)' = a_1 g' + 2a_2 g g' + 3a_3 g^2 g' + \cdots + n a_n g^{n-1} g' = [a_1 + 2a_2 g + 3a_3 g^2 + \cdots + n a_n g^{n-1}]g' = f'(g(x))g'(x)$$