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Math 542

Assigned: 2/17/2014 Due: 2/24/2014

Problem 14 Suppose for every $x \in G$ that $x^2 = e$. Prove that G is abelian.

proof Let $x, y \in G$ and by our definition of G, $x^2 = y^2 = e$.
Also $(xy)^2 = e = xyxy$. Left multiplying x and right multiplying y, we get $xey = x(xyxy)y$

This simplifies to $xy = x^2yxy^2 = eyxe = yx$ OR

$\therefore xy = yx \ \forall x, y \in G$. Thus G is abelian. $\square$

Problem 15 Suppose $H \subseteq G$ is a subgroup of index 2, i.e., $[G:H] = 2$.
Prove that it is a normal subgroup of G.

proof By the hypothesis, H has only two cosets of G relative to H.

One of these cosets is H itself, and the other must be the complement $G \setminus H$. OR

Let $g \in G$. If $g \in H$, then $gH = H = Hg$ so $gH = Hg$
If $g \notin H$, then $gH = G \setminus H$ and $Hg = G \setminus H$.
Thus $gH = Hg$ as well

$\therefore H$ commutes with every element of G and by definition H is normal. $\square$