Problem 11

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Problem 1 *(Wed Feb 12)*

(a) Prove that there are no simple groups of order either 575 *or* 272.

(b) For any prime p prove there are no simple groups of order $p(p-1)$ *or* $p(p+2)$.

(a) *Proof.* Since $575 = 5^2 \times 23$, by Sylow 3, $n(23) = 23k+1$ and $n(23)|25$. So $n(23) = 1$. By Sylow 2, we know G has a normal subgroup of order 23.
Since $272 = 2^4 \times 17$, by Sylow 3, $n(17) = 17k+1$ and $n(17)|2^4$. So $n(17) = 1$. By Sylow 2, we know G has a normal subgroup of order 17.

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(b) *Proof.* Since $(p, p-1) = 1$, by Sylow 3, $n(p) = pk+1$ and $n(p)|p-1$. So $n(p) = 1$. By Sylow 2, we know G has a normal subgroup of order p.
Suppose $p(p+2) = p^n m$, where $(p^n, m) = 1$. If $p \geq 3$, then $n = 1$ for $p \nmid 2$. By Sylow 3, $n(p) = pk+1$ and $n(p)|p+2$. So $n(p) = 1$. By Sylow 2, we know G has a normal subgroup of order p. If $p = 2$, then the order of G is 2^3. So G is a 2-group. Since every p-group has nontrivial center, G is not simple.