

## Instructions

The midterm exam will be in class on Tuesday March 21. It will cover chapters 11, 19-23.

The following problems are due in class at that time. Do not communicate with anyone else except me concerning these three problems.

Do not hand in this sheet. Show all work. Write clearly and carefully. If you are in doubt as to whether something should be proved either ask me or supply a proof.

1. Prove that  $1, \sqrt{2}, \sqrt{3}$  are linearly independent over the field of rational numbers,  $\mathbb{Q}$ .
2. Prove that  $[\mathbb{Q}(\sqrt{2} + \sqrt{3}), \mathbb{Q}] = 4$ .
3. Suppose that  $\mathbb{Q} \subseteq V \subseteq \mathbb{R}$  and  $V$  is a finite dimensional vector space over  $\mathbb{Q}$  and that for every  $x, y \in V$  that  $xy \in V$ . Prove that  $V$  is a field.

Show by example that we must assume that  $V$  is finite dimensional for this to be true.