Let p be a prime and G a finite group.

Definition 1 *Define group action* $T : G \times X \to X$, $orb(x)$, $stab(x)$, $[G : H]$, $Z(G)$, $C(a)$, $conj(a)$, *p-group, p-Sylow subgroup,* $N(H)$.

Proposition 2 *If* G *is group acting on a set* X*, then* $stab(x)$ *is a subgroup of* G *for any* $x \in X$ *and* $\{orb(x) : x \in X\}$ *partitions the set* X.

Theorem 3 *(Orbit-Stabilizer) Suppose* G *acts on* X*, then for any* $x \in X$

$$|orb(x)| = [G : stab(x)]$$

Theorem 4 *(Class equation)*

$$|G| = |Z(G)| + [G : C(a_1)] + [G : C(a_2)] + \cdots + [G : C(a_n)]$$

where $conj(a_1), conj(a_2), \ldots, conj(a_n)$ *are the nontrivial conjugacy classes of* G.

Corollary 5 *Every p-group has a nontrivial center, hence is not simple unless its isomorphic to* $\mathbb{Z}_p$.

Corollary 6 *Groups of order* p^2 *are abelian.*

Theorem 7 *(Sylow 1) If* G *is a finite group and* p^n *divides* $|G|$*, then there exists a subgroup* $H \subseteq G$ *with* $|H| = p^n$.

Theorem 8 *(Sylow 2) If* G *is a finite group,* H *a p-subgroup of* G*, and* P *a p-Sylow subgroup of* G*, then there exists* $a \in G$ *such that* $H \subseteq aPa^{-1}$.

Corollary 9 *Let* G *be a finite group such that* p *divides* $|G|$.

(a) Any p-subgroup of G *is contained in a p-Sylow subgroup of* G.

(b) Any two p-Sylow subgroups of G *are conjugates.*

(c) Any two p-Sylow subgroups of G *are isomorphic.*

(d) A p-Sylow subgroup is of G *normal iff it is the only p-Sylow subgroup of* G.

Theorem 10 *(Sylow 3) If* $|G| = p^n m$ *where* p *does not divide* m *and* $n(p)$ *is the number of p-Sylow subgroups of* G*, then:*

(a) $n(p) = [G : N(P)]$ *for any* P *a p-Sylow subgroup of* G,

(b) $n(p)$ *divides* m*, and*

(c) $n(p) = 1$ *mod* p