

1. Define ring, commutative ring, ring with 1, integral domain, field.
2. Give an example of a ring which is not commutative.
3. Give an example of a ring which is commutative but does not have a 1.
4. Give an example of a commutative ring with 1 which is not an integral domain.
5. Give an example of an integral domain which is not a field.
6. Give an example of finite field.
7. Prove: Let R be a ring and $a, b, c \in R$
 - (a) $a0 = 0 = 0a$
 - (b) $a(-b) = (-a)b = -(ab)$
 - (c) $(-a)(-b) = ab$
 - (d) $a(b - c) = ab - ac$
 - (e) $(b - c)a = ba - ca$and assuming R has 1:
 - (f) $(-1)a = -a$
 - (g) $(-1)(-1) = 1$
8. Define $R_1 \oplus R_2$
9. Define R' is a subring of R .
10. Prove: Every finite integral domain is a field.
11. Define ring homomorphism, ideal, factor ring, kernel.
12. Prove: If $\phi : R \rightarrow R'$ is a ring homomorphism, then $\ker(\phi)$ is an ideal in R and $\phi(R)$ is a subring of R' .
13. Define the factor ring R/I and the quotient map $\phi : R \rightarrow R/I$.
14. Prove: R/I is a ring and the quotient map is an onto ring homomorphism with kernel I .

15. Prove: If $\rho : R \rightarrow R'$ is a ring homomorphism, $I = \ker(\rho)$, then R/I is isomorphic to $\rho(R)$.
16. Define prime ideal and maximal ideal.
17. Prove: If R is a commutative ring with 1 and I is an ideal, then R/I is an integral domain iff I is prime ideal.
18. Prove: If R is a commutative ring with 1 and I is an ideal, then R/I is a field iff I is a maximal ideal.
19. Give an example of a prime ideal which is not maximal.
20. Prove: Every field contains an isomorphic copy of either the rationals, $\mathbb{Q}$, or $\mathbb{Z}_p$ for some prime p .
21. Prove: Any integral domain is isomorphic to a subring of a field. (Hint: Field of Quotients)
22. Define $F[x]$ and $F(x)$ where F is a field.
23. Define the degree of a polynomial, $\deg(f)$ for $f \in F[x]$.
24. Prove: $\deg(fg) = \deg(f) + \deg(g)$ where $f, g \in F[x]$
25. Prove: Euclidean Division Algorithm: For $f, g \in F[x]$ with g not the zero polynomial, there exists unique $q, r \in F[x]$ such that $f = qg + r$ and $\deg(r) < \deg(g)$.
26. Prove: For any field F , $f \in F[x]$, and $\alpha \in F$ if $f(\alpha) = 0$, then $f = (x - \alpha)g$ for some $g \in F[x]$.
27. Prove: For any polynomial $f \in F[x]$ if $\deg(f) = n \geq 0$, then f can have at most n distinct roots.
28. Prove: Any ideal in $F[x]$ is principal.
29. Define f is an irreducible polynomial in $R[x]$ for R a ring.
30. Give an example of a polynomial in $\mathbb{Z}[x]$ which is irreducible in $\mathbb{Q}[x]$ but reducible in $\mathbb{R}[x]$.
31. Give an example of a polynomial in $\mathbb{Z}[x]$ which is irreducible in $\mathbb{R}[x]$ but reducible in $\mathbb{C}[x]$.
32. What does the Fundamental Theorem of Algebra say?
33. Prove: If $f \in \mathbb{Z}[x]$ is irreducible in $\mathbb{Z}[x]$, then f is irreducible in $\mathbb{Q}[x]$.