

Name _____
Score _____
Grade _____

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M475 Exam 1
Oct 7, 1993

Put answers to problems 1-6 on your own paper.

1. A sequence of twelve positive integers sums to 103. Show that there must be two consecutive elements of the sequence which sum to at least 18. (Explain your answer.) (12 points)

2. How many three letter words over $\{A, B, C, \dots, Z\}$ do we need to choose to be sure that two of them agree in at least one place? (Note: letters may be repeated, e.g., BBA, XXX are allowed. BAB and CAD agree in the second place. ABC and BCA do not agree in any place.) (Explain your answer.) (11 points)

3. Suppose that each edge of the complete graph on 10 vertices, K_{10} , is colored either red or blue. Show that there must be either three vertices such that all of the edges connecting them are colored red (a red triangle), or four vertices such that all of the edges connecting them are colored blue (a blue square). (You may assume without proof that $K_6 \rightarrow (K_3, K_3)$.) (11 points)

4. How many paths are there from $(0, 0, 0)$ to $(3, 4, 5)$ if each move is one of the following: (11 points)

- $(x, y, z) \rightarrow (x + 1, y, z)$, or
- $(x, y, z) \rightarrow (x, y + 1, z)$, or
- $(x, y, z) \rightarrow (x, y, z + 1)$.

(Explain your answer.)

5. How many 8 letter words can be made by permuting the 8 letters: (11 points)

a, b, c, d, e, f, f, f

if no two f 's are to be next to each other? (Explain your answer.)

6. How many combinations of six taken from the numbers $\{1, 2, 3, \dots, 45\}$ are there such that at least two of the six numbers are consecutive? (Explain your answer.) (12 points)

In the following problems do **not** explain your answer.
Put your answers in the space provided.

7. In the lexicographical listing of all permutations of $\{1, 2, \dots, 9\}$ what permutation comes just after $\{2, 1, 3, 7, 9, 8, 6, 5, 4\}$? (5 points)
answer: _____

8. What are the mobile integers in $\overleftarrow{7} \overrightarrow{6} \overleftarrow{5} \overrightarrow{3} \overleftarrow{1} \overleftarrow{2} \overleftarrow{4} \overrightarrow{8} \overrightarrow{9}$? (6 points)
answer: _____
What is the next permutation after it (including the arrows)?
answer: _____

9. The sequence 6, 3, 2, 4, 0, 2, 1, 0 is an inversion sequence for a permutation of $\{1, 2, \dots, 8\}$. (6 points)
What is this permutation?
answer: _____
What is the inversion sequence for the permutation 2, 4, 7, 1, 8, 3, 6, 5?
answer: _____

10. The subsets of $\{x_5, x_4, x_3, x_2, x_1, x_0\}$ correspond to the numbers $0, 1, \dots, 63$ using the base 2 arithmetic scheme. What subset corresponds to the number 59? (5 points)
answer: _____

11. In the lexicographical listing of all 7-combinations of the set $\{1, 2, \dots, 15\}$ what 7-combination comes just after $\{5, 9, 10, 12, 13, 14, 15\}$? (5 points)
answer: _____

12. List all subsets of the set $\{x, y, z\}$ in such a way that if $A_1, A_2, \dots$, is your listing, then for every i the set A_{i+1} differs from the set A_i by either adding one element or taking one element away. (5 points)

answer: _____