Do any **five** of the following problems. Do not hand in this sheet. Do not hand in more than five problems.

Clearly state your assumptions before trying to prove something. State definitions clearly before using them in your proof. Define your notation before you use it. Do not assume what you are supposed to prove and then use that in your proof. Do both directions of an "if and only if" proof.

1. (a) How many subsets $A \subseteq \{1, 2, \ldots, 20\}$ are there of size six?

(b) How many subsets $A \subseteq \{1, 2, \ldots, 20\}$ are there of size six which do not contain a pair of successive integers? Equivalently $|A| = 6$ and for every $x, y \in A$ if $|x - y| \leq 1$ then $x = y$.

(c) How many subsets $A \subseteq \{1, 2, \ldots, 20\}$ are there of size six $|A| = 6$ such that for every $x, y \in A$ if $|x - y| \leq 2$ then $x = y$?

2. Show that if the edges of complete graph on 17 vertices are colored with three colors, then there three vertices such that the three edges connecting them have the same color. You may use without proof that $r(3, 3) = 6$.

3. Let $A_1, A_2, \ldots, A_n$ be a partition of a set X. Define the relation R on X by xRy iff x and y there is a k with $1 \leq k \leq n$ such that $x \in A_k$ and $y \in A_k$. Prove that R is an equivalence relation.

4. Prove that the only antichain of size 6 in the partial order of all subsets of $\{1, 2, 3, 4\}$ ordered by inclusion is the antichain of all 2-element subsets of $\{1, 2, 3, 4\}$.

5. Prove that

$$n! = \sum_{k=0}^{n} \binom{n}{k} D_{n-k}$$

6. Prove the following about the Fibonacci numbers:

f_n is divisible by 4 if and only if n is divisible by 6.

7. Prove that the number of onto functions from an n-set to a k-set is $k!\, S(n, k)$.

8. Let $\mathcal{A} = (A_1, A_2, \ldots, A_n)$ be a family of sets with an SDR. Let x be an element of A_1. Prove that there is an SDR containing x. Show by example that it may not be possible to find an SDR in which x represents A_1.