

MATH 475 Quiz 10/6/10

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The set of $A = \{1, 2, \dots, n\}$ is partitioned into 5 sets A_1, A_2, A_3, A_4, A_5 .

(a) What is the minimum n that guarantees that for some $k = 1, 2, 3, 4, 5$ that A_k contains at least k elements

$$|A_k| \geq k$$

In order for this to not work:

$$|A_1| \leq 0$$

$$|A_2| \leq 1$$

$$|A_3| \leq 2$$

$$|A_4| \leq 3$$

$$|A_5| \leq 4$$

$$|A| \leq 10$$

$|A_k| < k \rightarrow$ but adding 1 to any of this will make $|A_k| = k$ by pigeon hole principle

$$n = \left(\sum_{k=0}^{k-1} k \right) + 1 = \boxed{11}$$

(b) $n-1$ wouldn't work because it would allow each $|A_k|$ to be 1 less than k or

$|A_k| = k-1$ which doesn't satisfy the guarantee