

$$\frac{10}{10}$$

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Math 475
Take Home
Quiz 1

(a) How many ways to put 10 red balls + 8 blue balls into 4 numbered buckets?

(b) How many such ways have property that at least 1 bucket does not contain both red + blue balls?

solve: (a)

+5

$$S = \{ \langle x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4 \rangle : x_1+\dots+x_4=10, x_i \geq 0, y_1+\dots+y_4=8, y_i \geq 0 \}$$

$$|S| = \binom{10+3}{3} \times \binom{8+3}{3} = \underline{47,190}$$

(b).

+5

The complement of the question above is to find how many ways have property that all the buckets contain both red + blue balls.

$$\Leftrightarrow \{ \langle x_1+y_1, \dots, x_4+y_4 \rangle : x_1+\dots+x_4=10, x_i \geq 1, y_1+\dots+y_4=8, y_i \geq 1 \} \Rightarrow \{ \langle v_1+w_1, \dots, v_4+w_4 \rangle : v_1+\dots+v_4=6, v_i \geq 0, w_1+\dots+w_4=4, w_i \geq 0 \}$$

$$\Rightarrow \binom{6+3}{3} \times \binom{4+3}{3} = 2940$$

$$\text{The complement} = 47,190 - 2,940 = \underline{44,250}$$