

## Instructions

Do not hand in this sheet. Show all work. Write clearly and carefully.

1. At a party there are 10 men and 7 women.
  - (a) How many ways are there to form 7 couples consisting of one man and one woman?
  - (b) How many ways are there to form 5 couples consisting of one man and one woman?
  
2. The line segments joining 10 points are arbitrarily colored red or blue. Prove that there must exist three points such that the three line segments joining them are all red, or four points such that the six line segments joining them are all blue. Equivalently, prove that  $10 \rightarrow (3, 4)^2$  or  $r(3, 4) \leq 10$ .  
(You may assume without proof that  $6 \rightarrow (3)_2^2$  or equivalently  $r(3, 3) \leq 6$ .)
  
3. How many permutations of  $\{1, 2, 3, 4, 5\}$  have
  - (a) exactly 10 inversions?
  - (b) exactly 9 inversions?
  - (c) exactly 2 inversions?
  
4. Construct a partition of the subsets of  $\{1, 2, 3, 4\}$  into symmetric chains.
  
5. What is the coefficient of  $x^2y^3z$  in the expansion of

$$(2x - y + 4z)^6 ?$$

## Answers

1. a.  $\binom{10}{7} 7!$

1. b.  $\binom{10}{5} \binom{7}{5} 5!$

2. Fix any vertex  $v$ . There are two cases.

Case 1. There are 4 or more edges from  $v$  colored red.

Case 2. There are 6 or more edges from  $v$  colored blue.

3. 1,4,9

4. This is in the book on page 153.

5. -960