DAVID ESTES
M475

Hw due fri 04/16

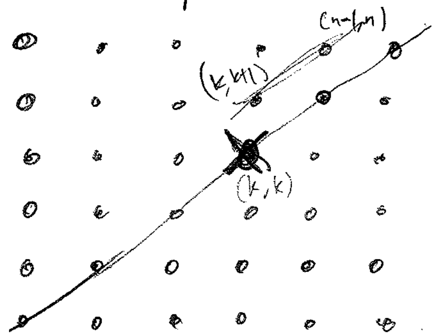
Ok

(41) Give a counting paths proof that satisfies $C_0 = 1 = C_1$

$$C_n = C_0 \cdot C_{n-1} + C_1 \cdot C_{n-2} + \dots + C_{n-1} C_0 = \sum_{k+j=n-1} C_k C_j$$

(j,j) not a path

Hint: look at last place before end visit diagonal (will be (k,k) ... not counting (n,n))



~~if at (4,4) and this is last place we hit on diagonal before (6,6) we see C_4 ways to arrive to 4 and then we must stay above diagonal so we must move diagonal up in a sense or take paths from (4,5) to (5,5) is C_1~~

~~so total paths from (0,0) to (~~

Say (k,k) is last place we hit on diagonal before (n,n). Then from (k,k) we must go up to $(k,k+1)$ and then take the number of paths at or above diagonal between points $(k,k+1)$ and $(n-1,n)$ then move to (n,n). This is like staying at or above the diagonal of an $n-(k+1)$ grid. so the number of paths is C_{n-1-k}. Since C_k ways to get to (k,k), the total paths from $(0,0)$ to (n,n) that make their last stop on the diagonal at (k,k) is $(C_k)(C_{n-1-k})$

Since the number of paths from $(0,0)$ to (n,n) is C_n and this counts every path that makes its last stop on the diagonal at (j,j) for some $0 \le j < n$. Call these 'C_j'. Then we see $C_n = \sum_{j=0}^{n-1} C_j$

Then from above: $C_n = \sum_{j=0}^{n-1} C_j = \sum_{k=0}^{n-1} (C_k)(C_{n-1-k}) = C_0 C_{n-1} + \dots + C_{n-1} C_0$

$$= \sum_{k+j=n-1} C_k C_j \quad \text{as desired.}$$