

## Math 475 HW

$$(38) a_{n+2} = a_{n+1} + 6a_n + n^2 \quad a_0 = 0, a_1 = 1$$

38 OK

$$x^2 = x + 6 \quad \text{homogeneous}$$

39 OK

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$a_n^h = \alpha 3^n + \beta (-2)^n$$

$$\text{Guess } An^2 + Bn + C \quad \text{particular}$$

$$(A(n+2)^2 + B(n+2) + C) = (A(n+1)^2 + B(n+1) + C) + 6(An^2 + Bn + C) + n^2$$

$$A(n^2 + 4n + 4) + B(n+2) + C = A(n^2 + 2n + 1) + B(n+1) + C + 6An^2 + 6Bn + 6C + n^2$$

$$An^2 + (4A+B)n + (4A+2B+C) = 7An^2 + n^2 + (2A+7B)n + (A+B+7C)$$

$$-6An^2 + (2A-6B)n + (3A+B-6C) = n^2$$

$$-6A = 1 \quad A = -\frac{1}{6}$$

$$2A - 6B = 0 \quad 6B = -\frac{1}{3} \quad B = -\frac{1}{18}$$

$$3A + B - 6C = 0 \quad -\frac{10}{18} = 6C \quad C = -\frac{5}{54}$$

$$a_n^p = -\frac{1}{6}n^2 - \frac{1}{18}n - \frac{5}{54}$$

$$a_n = \alpha 3^n + \beta (-2)^n - \frac{1}{6}n^2 - \frac{1}{18}n - \frac{5}{54}$$

$$0 = \alpha + \beta - \frac{5}{54} \quad \alpha + \beta = \frac{5}{54}$$

$$1 = 3\alpha - 2\beta - \frac{17}{54}$$

$$\frac{71}{54} = 3\alpha - 2\beta$$

$$\alpha = \frac{5}{54} - \beta$$

$$\frac{71}{54} = 3\left(\frac{5}{54} - \beta\right) - 2\beta$$

$$\frac{28}{27} = -5\beta \quad \beta = -\frac{28}{135}$$

$$\alpha + \frac{28}{135} = \frac{5}{54}$$

$$\alpha = \frac{3}{10}$$

$$a_n = \frac{3}{10} \cdot 3^n - \frac{28}{135} (-2)^n - \frac{1}{6}n^2 - \frac{1}{18}n - \frac{5}{54}$$

$$(39) a_{n+2} = a_{n+1} + 6a_n + 3^n \quad a_0 = 0 \quad a_1 = 1$$

Same homogeneous part as #38

$$a_n^h = \alpha 3^n + \beta (-2)^n$$

$$a_n^p = A n 3^n$$

$$(A(n+2)3^{n+2}) = (A(n+1)3^{n+1}) + 6(A n 3^n) + 3^n$$

$$A n 3^{n+2} + 2 \cdot 3^{n+2} = A n 3^{n+1} + A 3^{n+1} + 6 A n 3^n + 3^n$$

$$A n 3^2 + 2 \cdot 3^2 = A n \cdot 3 + 3A + 6 A n + 1$$

$$9 A n + 18A = 3 A n + 3A + 6 A n + 1$$

$$18A = 1 \quad A = \frac{1}{18}$$

$$a_n^p = \frac{1}{18} n 3^n$$

$$a_n = \alpha 3^n + \beta (-2)^n + \frac{1}{18} n 3^n$$

$$0 = \alpha + \beta \quad \alpha = -\beta$$

$$1 = 3\alpha + 2\beta + \frac{1}{5}$$

$$1 = -3\beta - 2\beta + \frac{1}{5} \quad \frac{4}{5} = -5\beta \quad \beta = -\frac{4}{25} \quad \alpha = \frac{4}{25}$$

$$a_n = \frac{4}{25} \cdot 3^n - \frac{4}{25} (-2)^n + \frac{1}{18} n 3^n$$