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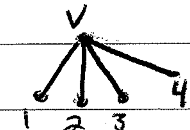
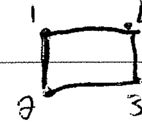
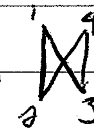
#26 Prove $K_8 \rightarrow (C_4, C_4)$ Taking 1 vertex & condidering 7 edges
There will at least 4 of one color (blue).
Take these four & condider connections between the vertices on the other end of the initial point.



Case 1 - All egdes between for points are 1 color
This automatically leaves the C_4

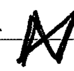


OR & C_4



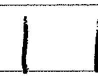
Case 2 - 5 are red 1 blue (matching color)
Choosing an edge it will either be an edge of

or

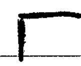


if it belongs to one, the other C_4 still exists of red.

Case 3 - 4 red 2 blue edges
4 subcases - 2 blue are ~~opp~~ "parralel" edges



2 blue are "perpendicular" edges



2 blue are outer edge and inner edge

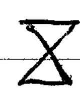
2 blue are inner edges

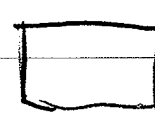
~~Case 1~~ Subcases 1, 2, 3 make blue C_4 with connections on the original point

(1→4, 4→0, 0→2, 2→4) or

(1→3, 3→0, 0→2, 2→4)

Subcases 1, 4 leave red C_4
opposite parrallels & inner two and all outer edges





Case 4 - 3 red and ~~3 [illegible] 3 blue~~ 3 blue

With the 3 blue you must have two blue that satisfy either subcase 2 or 3 of Case 3 so this is guaranteed a C_4 of blue.

From here any increase in blue edges leaves us satisfying At Least one subcase from Case 2. ✓ ∎