22 ok
23 ok

Math 475
HW # 12.

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22. <1>. How many binary relations R are there on the set $\{1, 2, ---, n\}$ which are reflexive?

If the relation is reflexive, we must include the n pairs of an element with itself but can choose other pairs arbitrarily. That gives us $\underline{2^{n^2-n}}$

<2> How many binary relations R are there on the set $\{1, 2, ---, n\}$ which are irreflexive?

The number of irreflexive relations is the same as that of reflexive relations, b/c if the relation is irreflexive, then we must exclude the n pairs of an element with itself but we can choose other pairs arbitrarily.
So $\underline{2^{n^2-n}}$

<3> How many binary relations R are there on the set $\{1, 2, ---, n\}$ which are symmetric?

If the relation is symmetric, we may choose for each of the $\binom{n}{2} = \frac{n^2-n}{2}$ unordered pairs of different elements (since if we include one ordered pair, we also include the symmetric pair) and seperately for the n pairs of an element with itself, giving $2^{n+\binom{n}{2}} = 2^{\frac{2n+n^2-n}{2}} = \underline{2^{\frac{n^2+n}{2}}}$

<4> How many binary relations R are there on the set $\{1, 2, ---, n\}$ which are symmetric and irreflexive?

According to <3>, we know there are $2^{n+\binom{n}{2}}$ symmetric binary relations.

For symmetric + irreflexive, we just need to subtract the reflexive ones from all the symmetric binary relations. There are n pairs of an element that can go with itself.

Therefore, $2^{n+\binom{n}{2}-n} = 2^{\binom{n}{2}} = \underline{2^{\frac{n^2-n}{2}}}$.

(23). Draw a Hasse diagram for the partial order:

$$(\{1,2,3,\cdots,32\}, \trianglelefteq).$$

where $x \trianglelefteq y$ iff x divides y.

It's drawn on the next page!

This is the best I can do!

Hasse Diagram