① CS475 20 ok
21 ok

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(2-12F)

20.

$A_1, A_2 \cdots A_2 \rightarrow$ All subsets of $\{1,2 \ldots n\}$ using increment and carry listing.

$N = 2^n$; $f: \{1, \ldots N\} \rightarrow \{1, \ldots N\}$

$$\boxed{f(k) = (2^n + 1) - k}$$

For $n = 4$

$A_1 = \{\ \}$
$A_2 = \{1\}$
$A_3 = \{2\}$
$A_4 = \{1,2\}$
$A_5 = \{3\}$
$A_6 = \{1,3\}$
$A_7 = \{2,3\}$
$A_8 = \{1,2,3\}$
$A_9 = \{4\}$
$A_{10} = \{1,4\}$
$A_{11} = \{2,4\}$
$A_{12} = \{1,2,4\}$
$A_{13} = \{3,4\}$
$A_{14} = \{1,3,4\}$
$A_{15} = \{2,3,4\}$
$A_{16} = \{1,2,3,4\}$

$A_{(2^n+1)-k} = \bar{A}_k$

This holds good for all A_k s.

Hence Proved.

$f(16) = (16+1) - 16 = 1$

$\therefore A_{16} = \bar{A}_1$ [which is correct]

$A_5 = \bar{A}_{12}$ (which is correct)

$f(5) = (16+1) - 5 = 12$

Let's say we generate the permutations of $\{1,2\ldots n\}$

21. by ~~[illegible]~~ sequence of adjacent swaps.

In this case, the permutations p_k & p_{k+1} differ by a single swap between adjacent elements. ~~Let~~ The numbers in a permulation are denoted by $a_1, a_2 \ldots a_n$

Let's consider the inversion sequences $\vec{i}_1, \vec{i}_2 \ldots \vec{i}_N$ for the permutations $p_1, p_2 \ldots p_N$ (Permutations are obtained by adjacent swaps.)

Now, consider any two inversion sequences $\vec{i}_k$ and $\vec{i}_{k+1}$.

Let say p_{k+1} is obtained by swapping l^{th} element with $l+1^{th}$ element in ~~[illegible]~~ p_k. $(1 \leq l \leq n)$

$\Rightarrow$ (1) the numbers of elements greater than the elements ~~[illegible]~~ $(a_1, a_2 \ldots a_{l-1})$ is not affected by swapping the elements at position l with the element at position $l+1$

$\Rightarrow$ the inversion sequence $\vec{i}_{k+1}$ resembles i_k from $i_{k,1}$ to $i_{k,l-1}$

~~(i.e) $i_{k,m} = i_{k+1,m}$, $1 \leq l \leq$~~

~~for all l~~

(i.e) $i_{k,m} = i_{k+1,m}$, $1 \leq m \leq l-1$.

(2) Also, The numbers of elements greater than the elements $a_{l+1}, a_{l+2} \dots a_n$ is not affected by swapping the elements at position l with the element at position $l+1$

$\Rightarrow$ the inversion sequence $\vec{i}_{k+1}$ resembles $\vec{i}_k$ from $\vec{i}_{k,l+2}$ to $i_{k,n}$

(i.e) $i_{k,m} = i_{k+1,m}$, $l+2 \leq m \leq n$.

Among the elements a_l & a_{l+1} , one of them is greater than the other $\Rightarrow$ If a_l is smaller, $i_{k+1,l}$ will be different from $i_{k,l}$ $\Rightarrow$ change in one co-ordinate alone

case(i) If $a_l < a_{l+1}$

then, $\vec{i}_{k+1,l} = \vec{i}_{k,l} + 1$

case(ii): if $a_l > a_{l+1}$

$\vec{i}_{k+1,l} = \vec{i}_{k,l} - 1$

④ [Page 3 in back! Sorry]

$\Rightarrow$ Combining case(i) and case(ii),

$$|i_{k+1,l} - i_{k,l}| = 1.$$

Permutations of S_4	Corresponding inversion seq
1 2 3 4	0 0 0 0
1 2 4 3	0 0 1 0
1 4 2 3	0 1 1 0
4 1 2 3	1 1 1 0
4 1 3 2	1 2 1 0
1 4 3 2	0 2 1 0
1 3 4 2	0 2 0 0
1 3 2 4	0 1 0 0
3 1 2 4	1 1 0 0
3 1 4 2	1 2 0 0
3 4 1 2	2 2 0 0
4 3 1 2	2 2 1 0
4 3 2 1	3 2 1 0
3 4 2 1	3 2 0 0
3 2 4 1	3 1 0 0
3 2 1 4	2 1 0 0

5

2 3 1 4	2 0 0 0
2 3 4 1	3 0 0 0
2 4 3 1	3 0 1 0
4 2 3 1	3 1 1 0
4 2 1 3	2 1 1 0
2 4 1 3	2 0 1 0
2 1 4 3	1 0 1 0
2 1 3 4	1 0 0 0

The inversion sequences i_k & i_{k+1} differ by a change in one co-ordinate and the change is by ± 1

$\Rightarrow |i_{k,l} - i_{k+1,l}| = 1$

[All the above statements are valid given that the inversion sequences are ~~obt~~ obtained for permutations obtained by sequence of adjacent swapping]

Math 475 hw 20, 21

Sean Bauer

20 ok
21 ok

20) Using increment & carry from right

$A_1 = 0000 = \{\ \}$

$A_2 = 0001 = \{4\}$

$A_3 = 0010 = \{3\}$

$\vdots$

$A_{15} = 1110 = \{1,2,3\}$

$A_{16} = 1111 = \{1,2,3,4\}$

$$\underline{f(k) = N+1-k}$$

$$A_{f(k)} = \overline{A_k}$$

Check

$A_1 = \overline{A_{16}}$ $\quad \{\ \}$ vs. $\{1,2,3,4\}^c$ ✓

$A_5 = A_{12}$ $\quad \{2,4\}$ vs. $\{1,3\}$ ✓

etc.

21) Build the listing of I_n as follows:

Base case:

$n=1$

0

Take the sequence of $(n-1)$ I's, and list them once in order, then in reverse order, then in order, etc. until you have listed the sequence n times. Then, append $(n-1)!$ 0's, then $(n-1)!$ 1's, etc. until $(n-1)!$ $(n-1)$'s.

Show.

$n=1$

0

$n=2$

00
10

$n=3$

000
010] reg order
110
100] rev.
200
210] reg

gray code

n=4	Permutations
0000	1234
0010	1243
0110	1423
0100	1324
0200	1342
0210	1432
1210	4132
1200	3142
1100	3124
1110	4123
1010	2143
1000	2134
2000	2314
2010	2413
2110	4213
2100	3214
2200	3412
2210	4312
3210	4321
3200	3421
3100	3241
3110	4231
3010	2431
3000	2341

List of S_4 Property

Each S_i & S_{i+1} differ by a single swap of 2 values.