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Math 475
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14) The hours on a clock face $\{1, 2, \ldots, 12\}$ are randomly permuted. Prove that there must be 3 hours in a row whose sum is at least 20.

Hint: Calculate $(x_1+x_2+x_3)+(x_2+x_3+x_4)+\ldots+(x_{11}+x_{12}+x_1)+(x_{12}+x_1+x_2)$

Pf: $\langle x_1, x_2, \ldots, x_{12}\rangle$ permutations of $\langle 1, 2, \ldots, 12\rangle$

We consider the sum

(*) $(x_1+x_2+x_3)+(x_2+x_3+x_4)+\ldots+(x_{11}+x_{12}+x_1)+(x_{12}+x_1+x_2)$

Since each x_i, $1 \le i \le 12$, appears exactly 3 times in the sum, corresponding to the element j, where $j \in \{1, 2, \ldots, 12\}$, we have 3 copies of each integer from 1 to 12 (inclusive), the sum (*) corresponds to $3(1+2+\ldots+11+12) = 234$ taking "average", we have $\frac{234}{12} = 19.5$

If each 3 hours in a row sums up to ≤ 19,

then we will have $19 \cdot 12 = 228 < 234$

Thus, by Pigeonhole Principle,

there must be 3 hours in a row whose sum is ~~at least 20~~ greater than 19, that is at least 20.

$\blacksquare$